

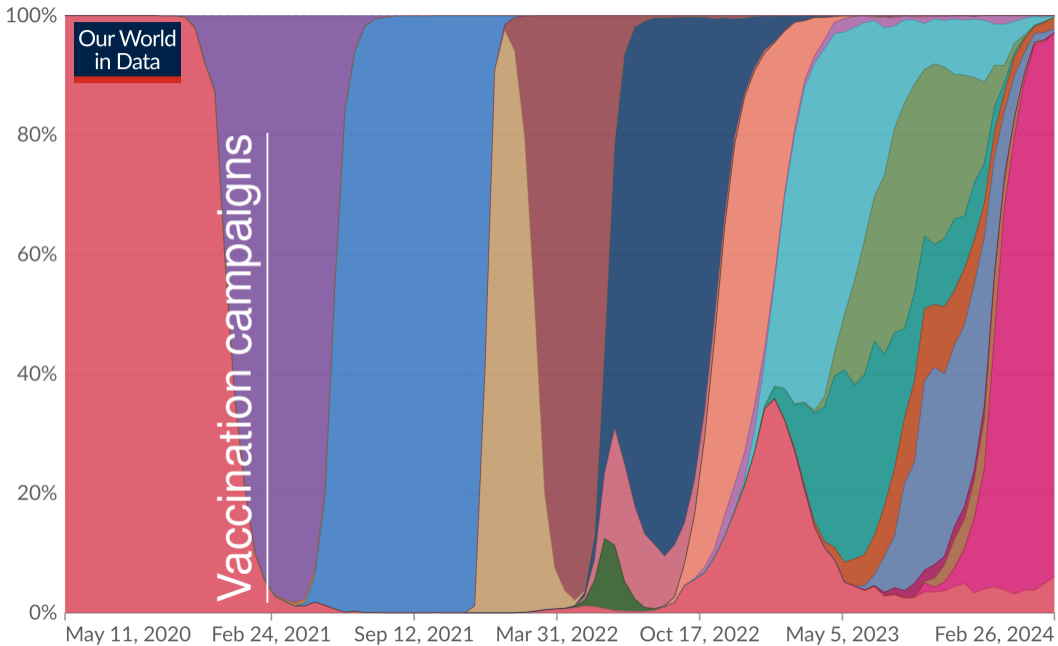
# Modelling vaccine escape

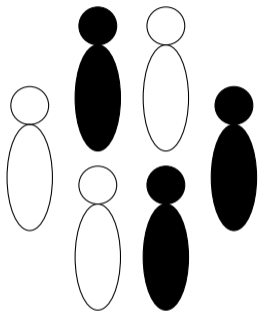
Maria A. Gutierrez (mag84) and Julia R. Gog

Dept. of Applied Mathematics and Theoretical Physics  
(DAMTP)

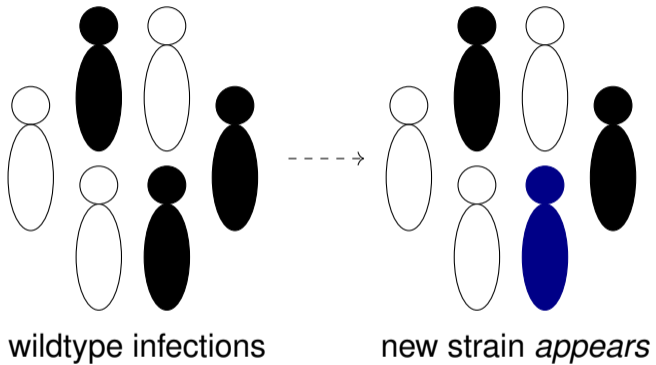


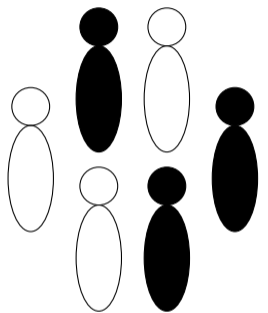




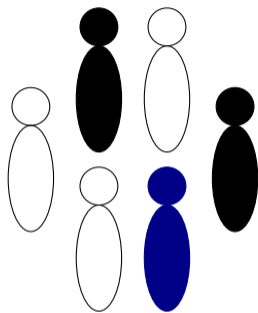


wildtype infections

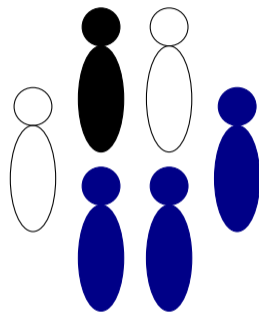




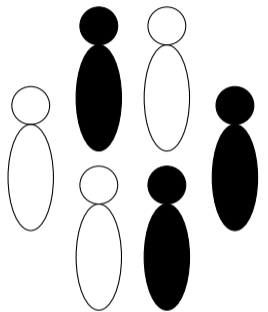
wildtype infections



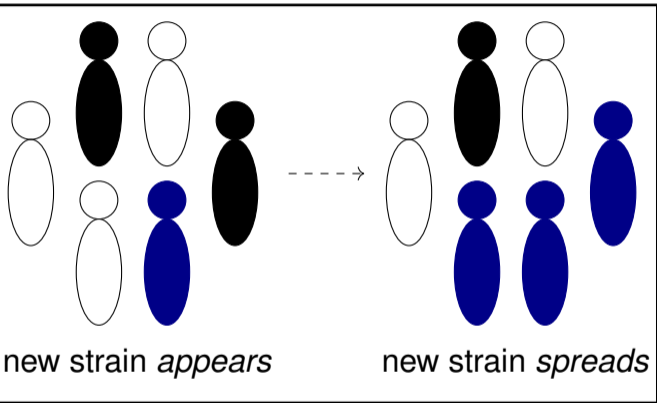
new strain *appears*



new strain *spreads*

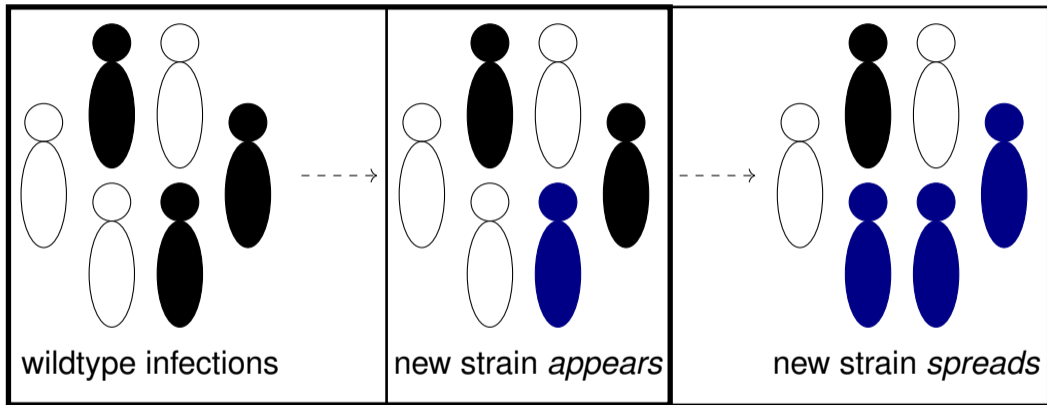


wildtype infections

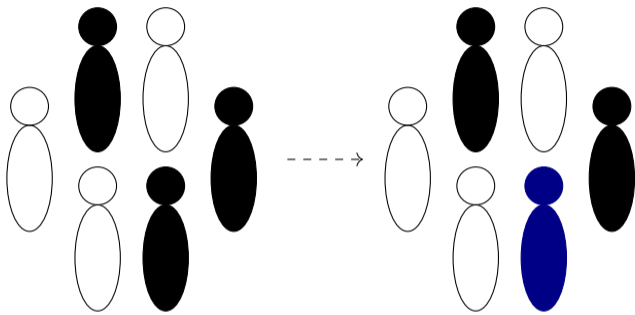


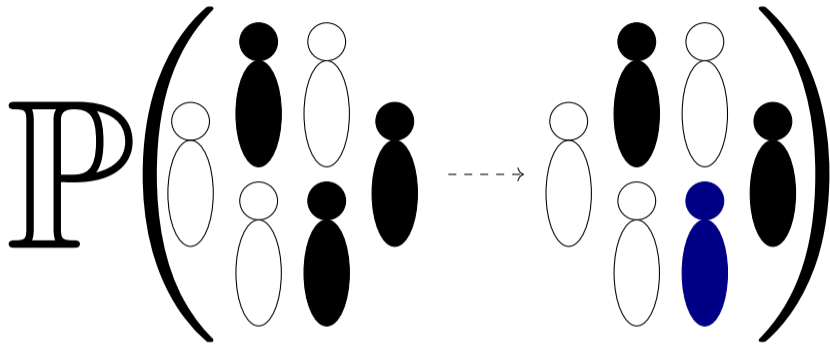
new strain *appears*

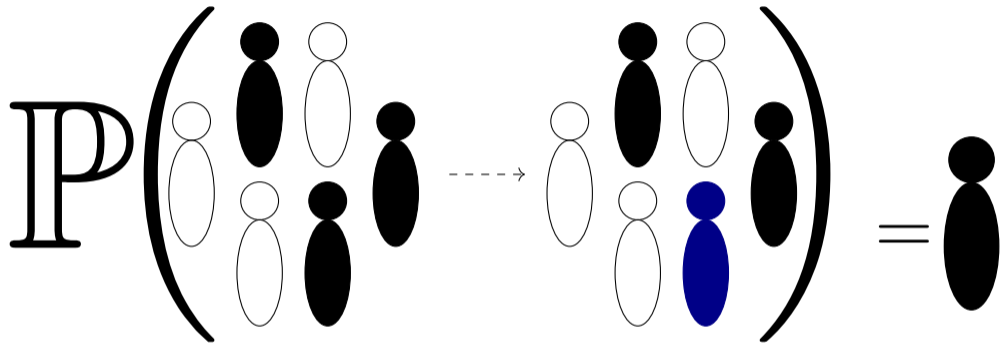
new strain *spreads*



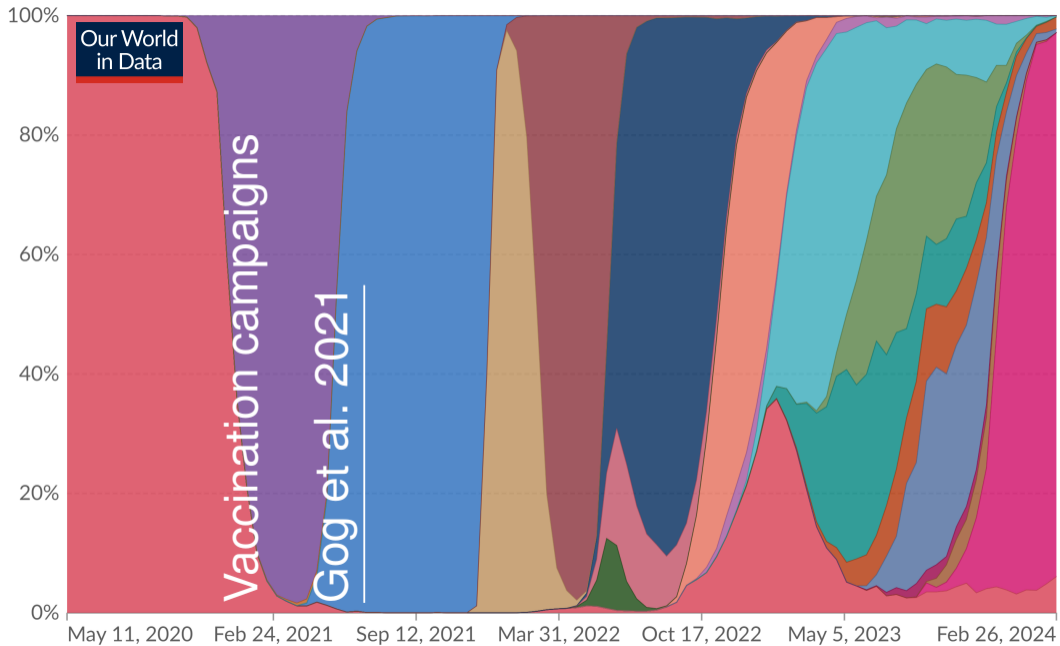


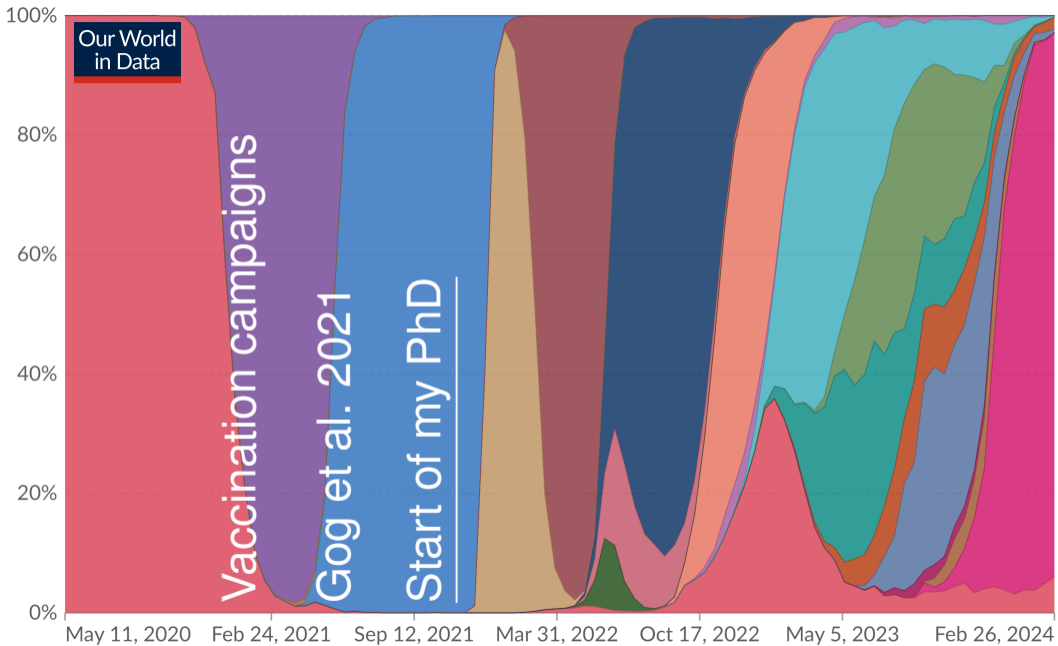


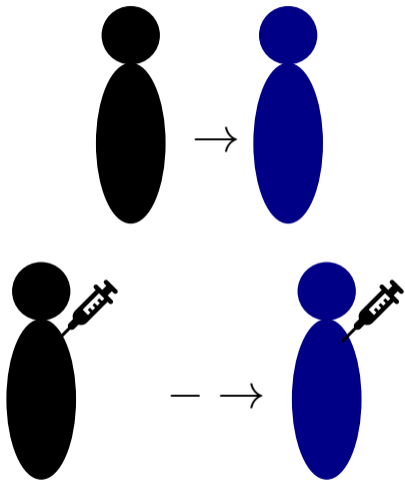


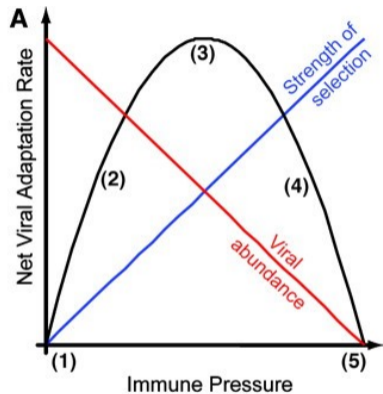
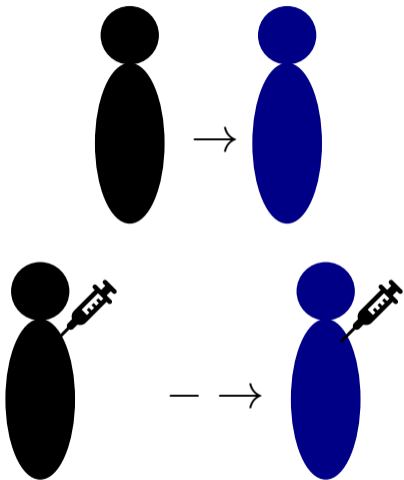


Hartfield et al. 2014, Gog et al. 2021,  
Rella et al. 2021, Saad-Roy et al. 2021









Grenfell *et al.*, Science 2004

$$\mathbb{P} \left( \left( \begin{array}{cccc} \text{white} & \text{black} & \text{white} & \text{black} \\ \text{white} & \text{black} & \text{white} & \text{black} \end{array} \right) \xrightarrow{\text{dashed arrow}} \left( \begin{array}{cccc} \text{white} & \text{black} & \text{white} & \text{black} \\ \text{white} & \text{black} & \text{blue} & \text{black} \end{array} \right) \right) = \text{black}$$

The diagram illustrates a probability calculation. On the left, a large letter 'P' is followed by a large left parenthesis. Inside the parenthesis, a group of eight stylized human figures is arranged in two rows of four. The top row contains a white figure, a black figure, a white figure, and a black figure. The bottom row contains a white figure, a black figure, a white figure, and a black figure. A dashed arrow points from this group to a second group of eight stylized human figures, also arranged in two rows of four. The top row is identical to the first group. The bottom row contains a white figure, a black figure, a blue figure, and a black figure. To the right of this second group is a large right parenthesis, followed by an equals sign and a single black stylized human figure.



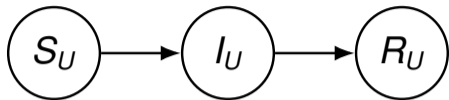
$$\mathbb{P}\left(\begin{array}{cccc} \bullet & \bullet & \circ & \bullet \\ \circ & \bullet & \circ & \bullet \\ \circ & \bullet & \bullet & \bullet \\ \circ & \bullet & \bullet & \bullet \end{array} \rightarrow \begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \circ & \bullet & \circ & \bullet \\ \circ & \bullet & \bullet & \bullet \\ \circ & \bullet & \bullet & \bullet \end{array}\right) = \bullet + \theta_E \bullet \text{ (with syringe)}$$

The diagram illustrates a transition in a population of 12 individuals. On the left, inside large parentheses, is a 4x4 grid of icons. The top row contains a black male, a black female, a white female, and a black male. The second row contains a white female, a black male, a white female, and a black male. The third row contains a white female, a black male, a black male, and a black male. The bottom row contains a white female, a black male, a black male, and a black male. A dashed arrow points to the right, where the same 4x4 grid is shown, but with the icon in the second row, third column (a black male) now colored blue. To the right of this grid is an equals sign, followed by a single black male icon, a plus sign, the Greek letter  $\theta_E$ , another plus sign, and a black male icon with a syringe icon on its back.

$$\mathbb{P} \left( \begin{array}{c} \text{[Group of 7 people: 4 black, 3 white]} \\ \text{---} \\ \text{[Group of 7 people: 4 black, 1 blue, 2 white]} \end{array} \right) = \text{[1 black person]} + \theta_E \text{[1 black person with syringe]}$$

$$P(t) = I_U(t) + \theta_E I_V(t)$$

Unvaccinated fraction  $(1 - c)$

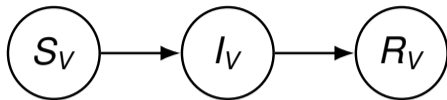


$$\dot{S}_U = -S_U\lambda(t)$$

$$\dot{I}_U = S_U\lambda(t) - I_U$$

$$S_U(0) = (1 - c)$$

Vaccinated fraction  $c$



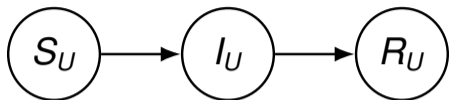
$$\dot{S}_V = -S_V\lambda(t)$$

$$\dot{I}_V = S_V\lambda(t) - I_V$$

$$S_V(0) = c\theta_s$$

$$\lambda = R_0(I_U + \theta_I I_V)$$

Unvaccinated fraction  $(1 - c)$

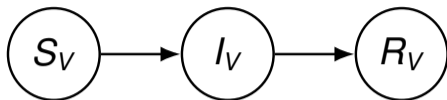


$$\dot{S}_U = -S_U \lambda(t)$$

$$\dot{I}_U = S_U \lambda(t) - I_U$$

$$S_U(0) = (1 - c)$$

Vaccinated fraction  $c$



$$\dot{S}_V = -S_V \lambda(t)$$

$$\dot{I}_V = S_V \lambda(t) - I_V$$

$$S_V(0) = c\theta_S$$

$$\lambda = R_0(I_U + \theta_I I_V)$$

$\forall t$

$$(1 - c)^{-1}(S_U, I_U, R_U) = (c\theta_S)^{-1}(S_V, I_V, R_V)$$

$$(S, I) := (1 - c)^{-1}(S_U, I_U) = (c\theta_S)^{-1}(S_V, I_V)$$

$$R_e = R_0(1 - c(1 - \theta_S\theta_I))$$

$$S(\infty) = -W(-R_e e^{-R_e}) / R_e$$

$$(S, I) := (1 - c)^{-1}(S_U, I_U) = (c\theta_S)^{-1}(S_V, I_V)$$

$$R_e = R_0(1 - c(1 - \theta_S\theta_I))$$

$$S(\infty) = -W(-R_e e^{-R_e}) / R_e$$

$$\int_0^\infty P(t) dt = \int_0^\infty (I_U + \theta_E I_V) dt$$

$$P = (1 - c + \theta_S\theta_EC) \left( 1 + \frac{1}{R_e} W(-R_e e^{-R_e}) \right)$$

$P(c)/P(0)$ : escape pressure

10

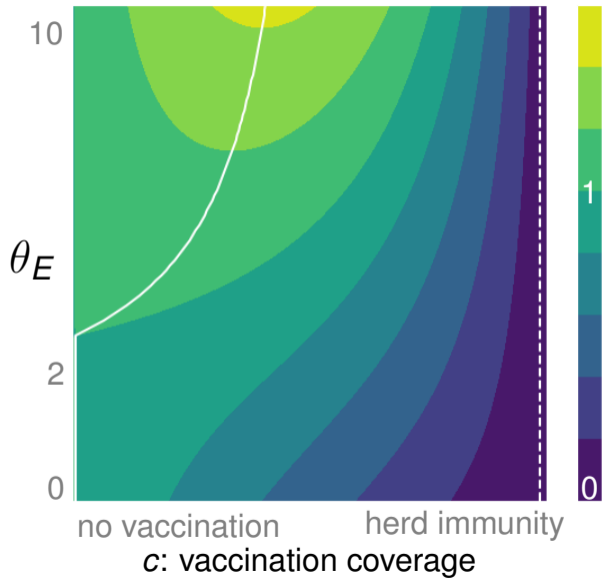
$\theta_E$

2

0

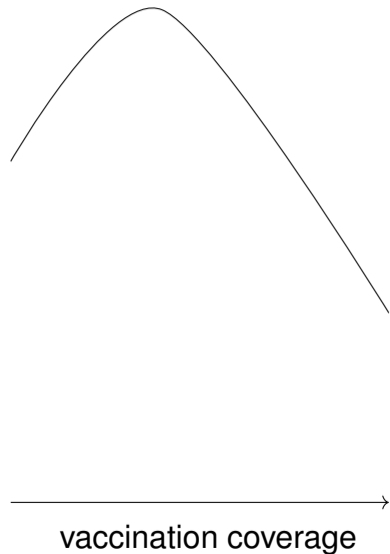
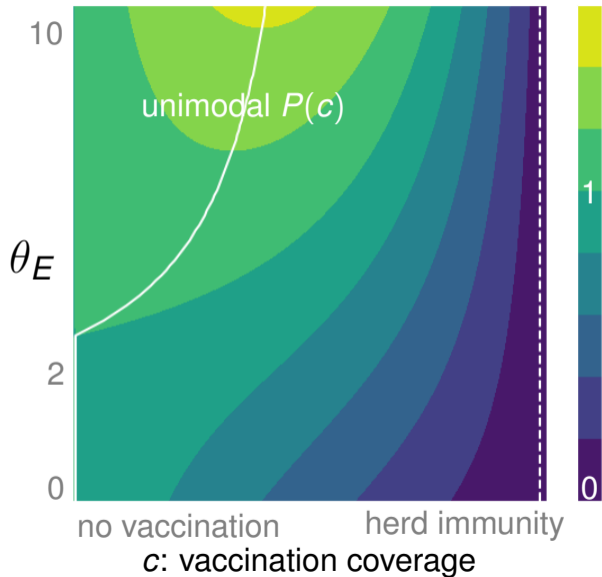
no vaccination      herd immunity  
 $c$ : vaccination coverage

$P(c)/P(0)$ : escape pressure

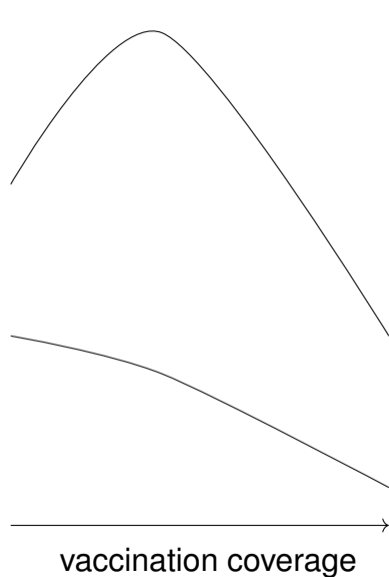
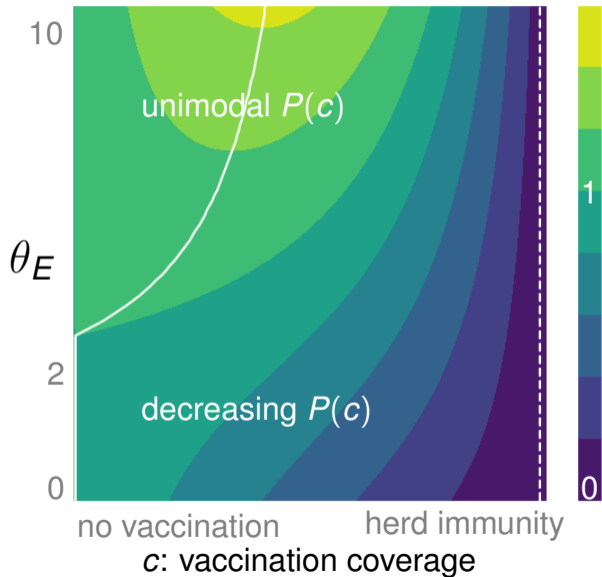




$P(c)/P(0)$ : escape pressure

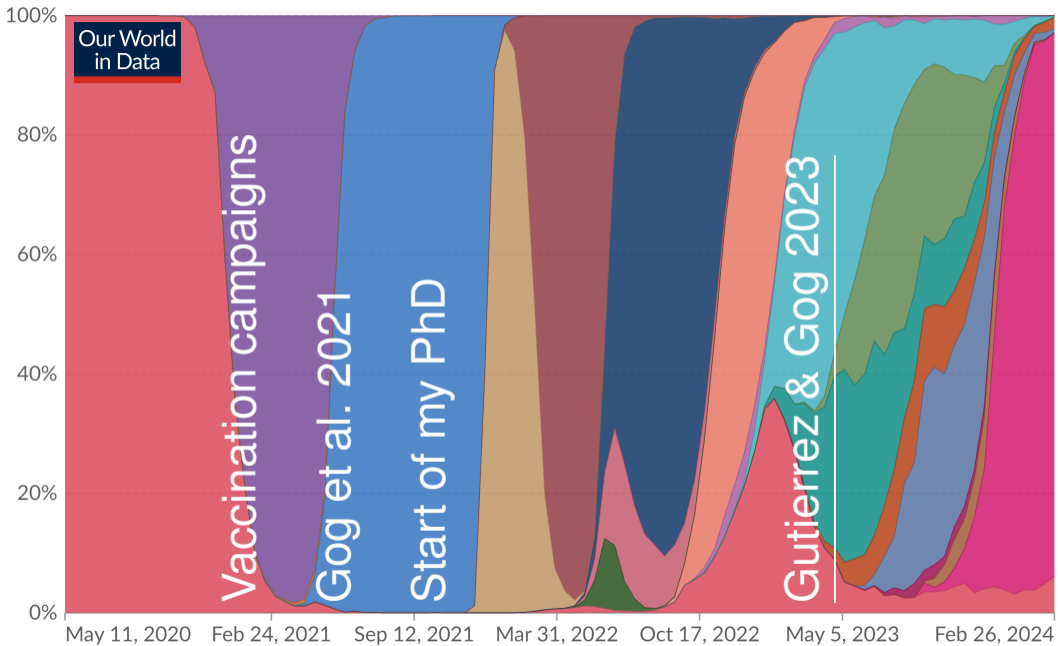


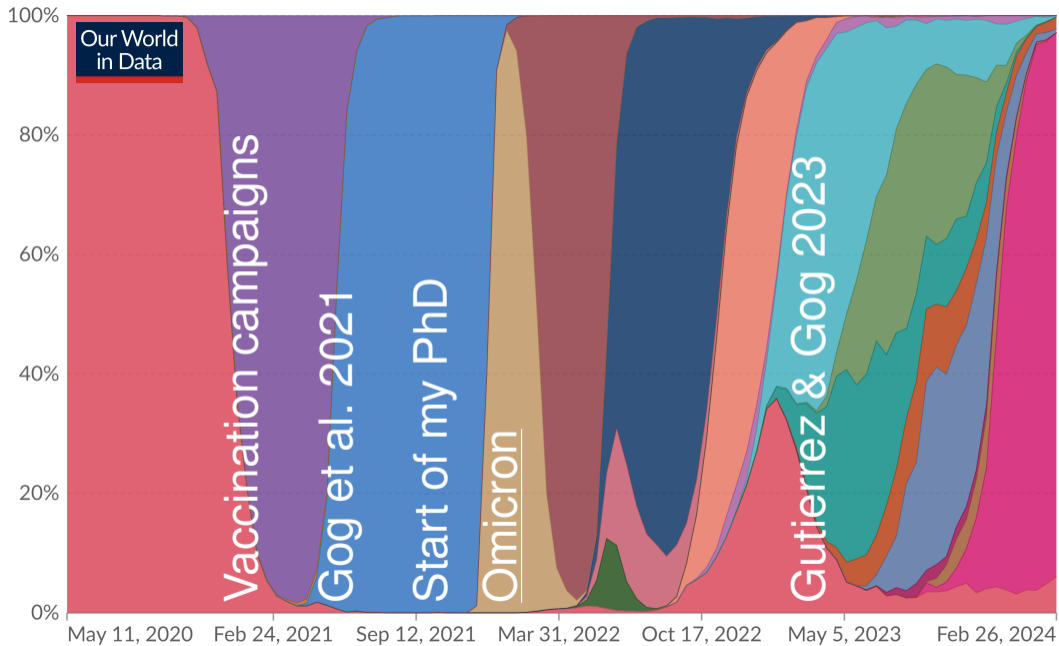
$P(c)/P(0)$ : escape pressure

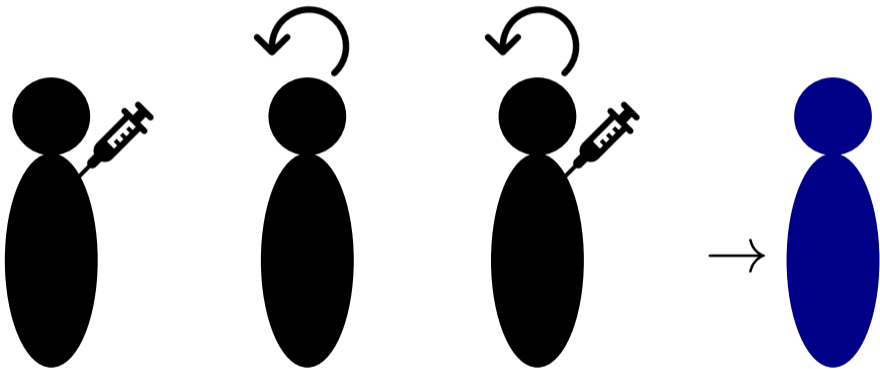




$\theta$  *E*







$$\mathbb{P}\left(\begin{array}{c} \text{[Group of 7 people: 3 black, 4 white]} \\ \text{---} \\ \text{[Group of 7 people: 3 black, 1 blue, 3 white]} \end{array}\right) = \text{[1 black person]} + \theta_E \text{[1 black person with syringe]}$$

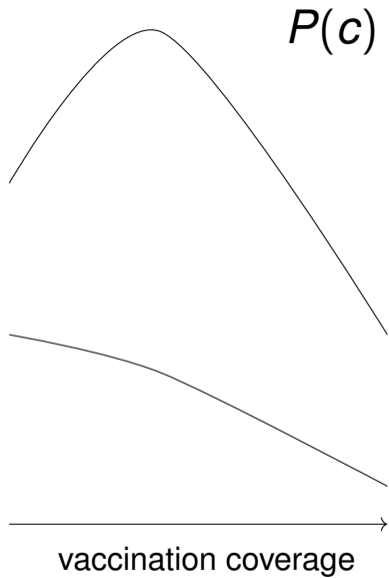
$$\mathbb{P} \left( \begin{array}{c} \text{[Group of 7 people: 2 white, 3 black, 2 black]} \\ \text{--->---} \\ \text{[Group of 7 people: 2 white, 1 black, 1 blue, 3 black]} \end{array} \right) = \text{[1 black]} + \theta_E \text{[1 black with syringe]} + \theta_E \text{[1 black with arrow]} + \theta_E \text{[1 black with syringe and arrow]}$$

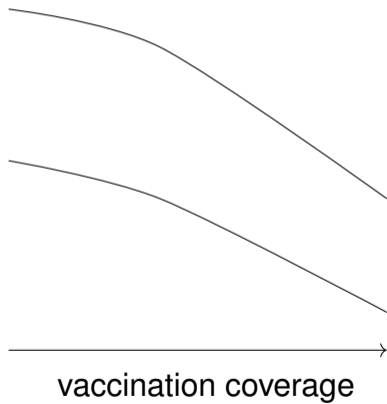
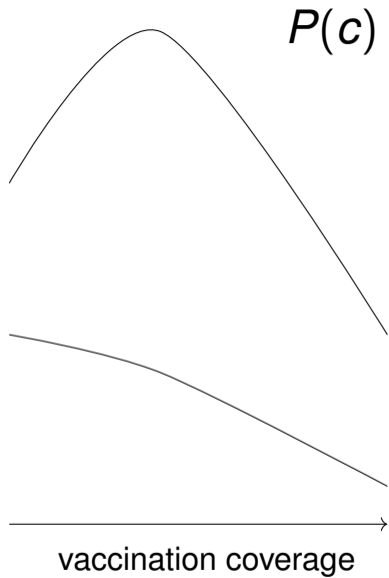


$$\mathbb{P} \left( \begin{array}{c} \text{[Group of 6 people: 2 white, 2 black, 1 black, 1 black]} \\ \text{---} \\ \text{[Group of 6 people: 2 white, 1 black, 1 blue, 1 black]} \end{array} \right) = \text{[1 black]} + \theta_E \text{[1 black with syringe]} + \theta_E \text{[1 black with arrow]} + \theta_E \text{[1 black with syringe and arrow]}$$

$$P = \left[ 1 - c + \theta_E \left( \frac{\theta'_S(1 - R_0^{-1}) + c(\theta_S - \theta'_S)}{1 - \theta'_S} \right) \right] \left( 1 + \frac{W[-re^{-r}]}{r} \right)$$

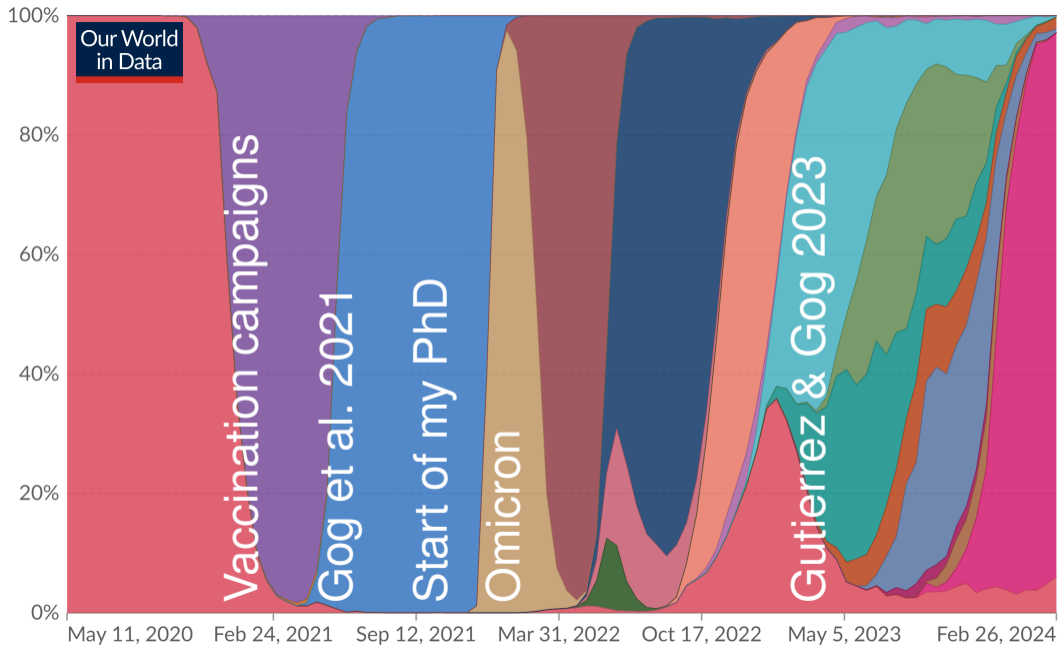
$$r = (R_e - \theta'_S)/(1 - \theta'_S)$$







$\theta_E, \theta_S, \theta'_S$





$$\mathbb{P}\left(\begin{array}{c} \text{[Group of 7 people: 3 black, 4 white]} \\ \text{---} \\ \text{[Group of 7 people: 3 black, 1 blue, 3 white]} \end{array}\right) = \text{[1 black person]} + \theta_E \text{[1 black person with syringe]}$$

$$\mathbb{P} \left( \begin{array}{c} \text{[Group of 7 people: 4 black, 3 white]} \\ \text{---} \\ \text{[Group of 7 people: 4 black, 1 white, 1 blue]} \end{array} \right) = \text{[1 black]} + \theta_E \text{[1 black with syringe]} + p \text{[1 black with exclamation mark]} + p\theta_E \text{[1 black with exclamation mark and syringe]}$$

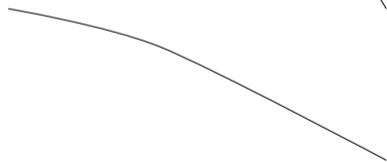
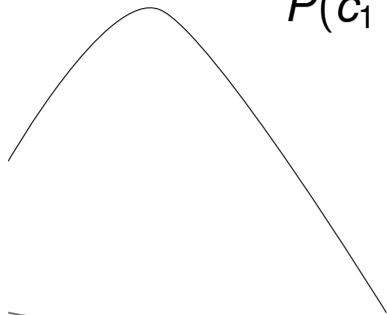


$$\mathbb{P} \left( \begin{array}{c} \text{[Initial State]} \end{array} \xrightarrow{\text{---}} \begin{array}{c} \text{[Final State]} \end{array} \right) = \text{[Initial State]} + \theta_E \text{[Vaccinated]} + p \text{[Infected]} + p\theta_E \text{[Vaccinated + Infected]}$$

The diagram illustrates a transition between two states of a population. The initial state (left) shows a group of seven individuals: two white, two black, and three black. A dashed arrow points to the final state (right), where the group consists of two white, one black, one blue, and three black individuals. The blue individual represents a vaccinated person. The equation to the right of the arrow shows the composition of the final state as a sum of terms: a single black individual, plus  $\theta_E$  times a vaccinated individual (black with a syringe), plus  $p$  times a black individual, plus  $p\theta_E$  times a vaccinated individual with a syringe.

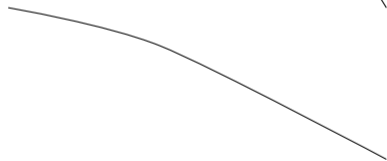
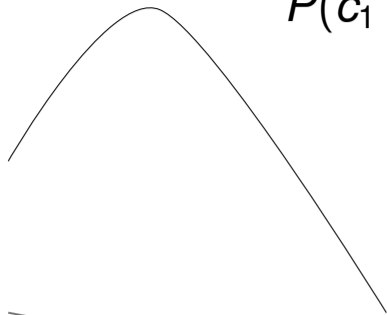
$$P(c_1, c_2)$$

$$P(c_1 + c_2)$$

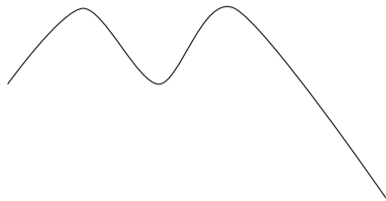


vaccination coverage

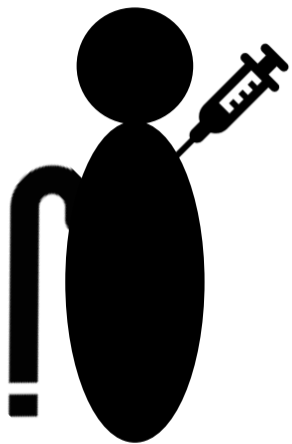
$$P(c_1 + c_2)$$



vaccination coverage



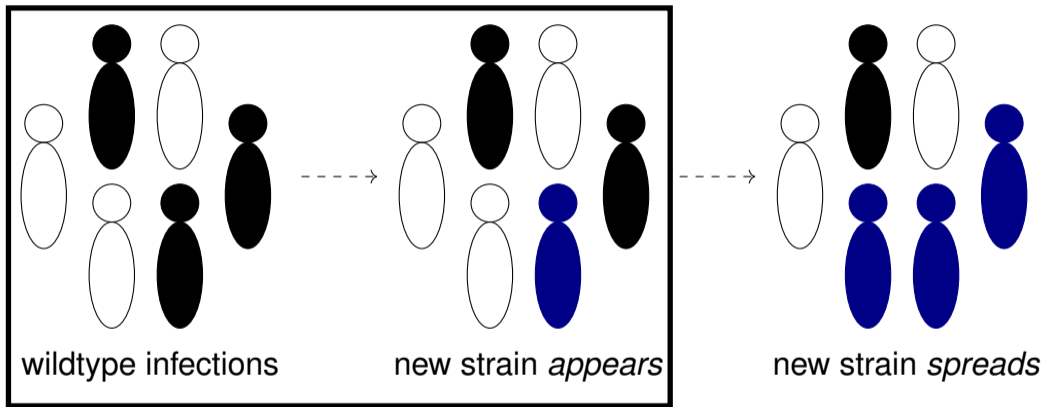
vaccination coverage

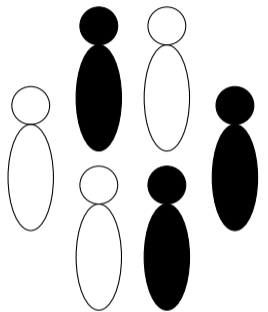


$\theta_E, \rho$

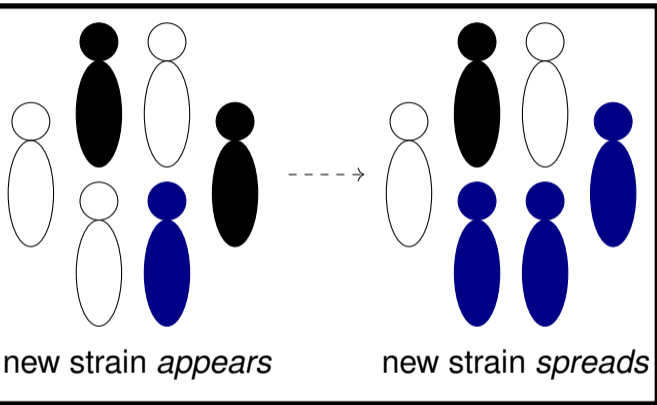


$\theta_E$



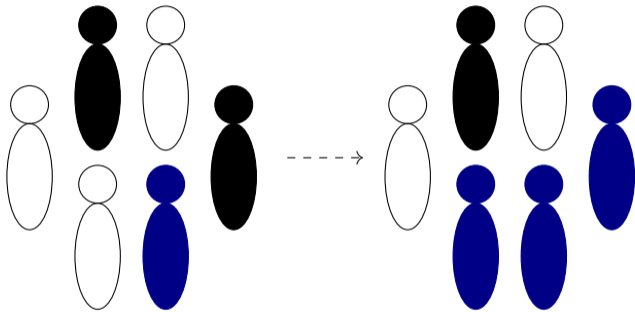


wildtype infections

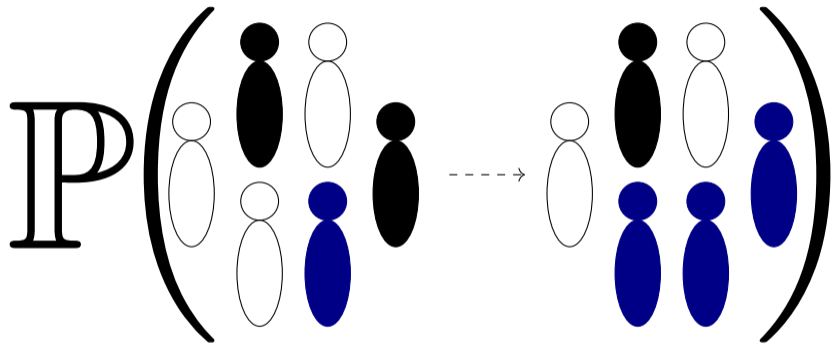


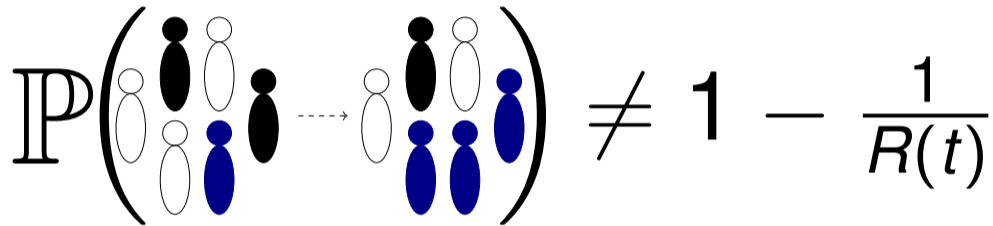
new strain *appears*

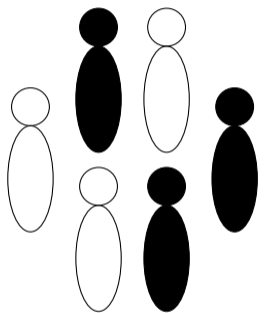
new strain *spreads*



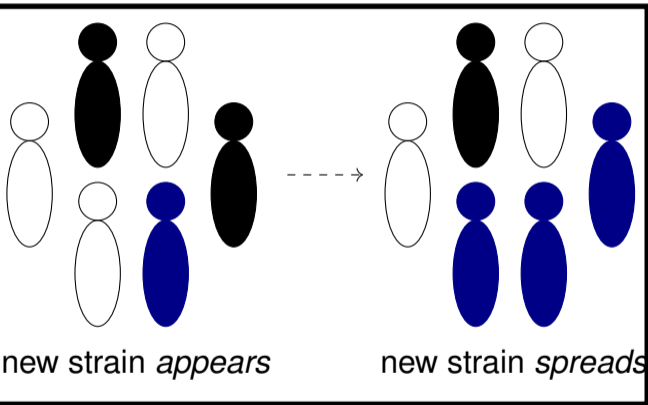




$$\mathbb{P}\left(\begin{array}{c} \text{Initial state} \xrightarrow{\text{Transition}} \text{Final state} \end{array}\right) \neq 1 - \frac{1}{R(t)}$$


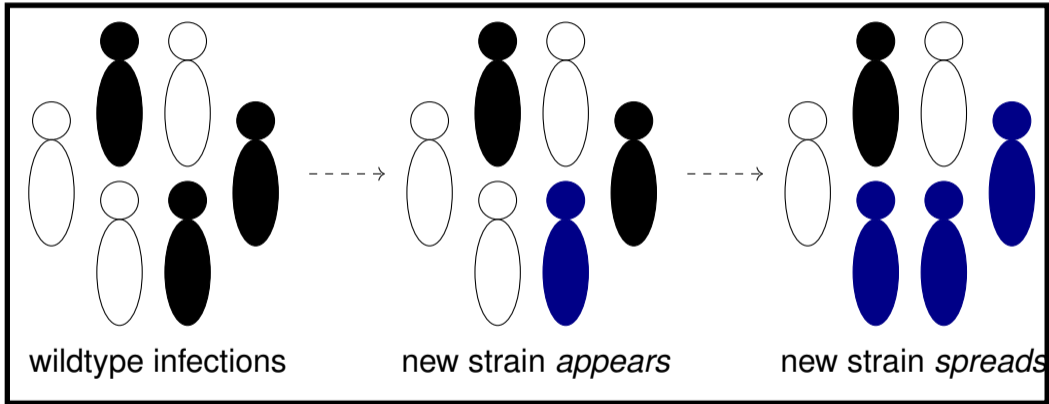


wildtype infections

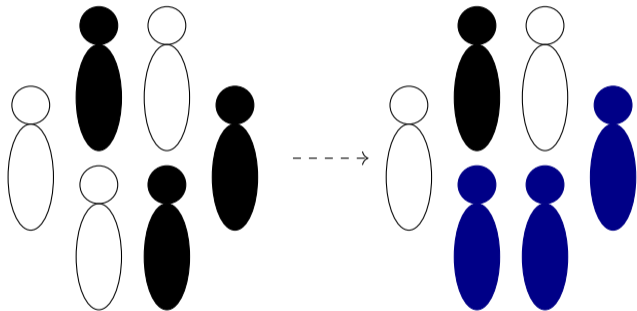


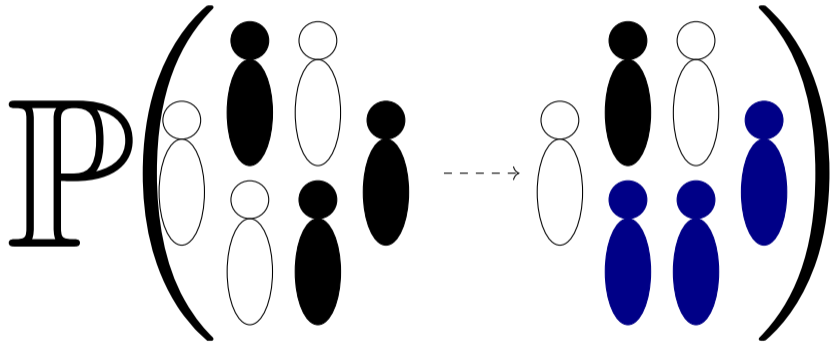
new strain *appears*

new strain *spreads*









$$\mathbb{P} \left( \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \quad \text{Black} \end{array} \xrightarrow{\text{dashed}} \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Blue} \quad \text{Blue} \end{array} \right) = \\
 \mathbb{P} \left( \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \quad \text{Black} \end{array} \xrightarrow{\text{dashed}} \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Blue} \quad \text{Black} \end{array} \right) \times \mathbb{P} \left( \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \quad \text{Black} \end{array} \xrightarrow{\text{dashed}} \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Blue} \quad \text{Blue} \end{array} \right)$$