Vaccine Escape

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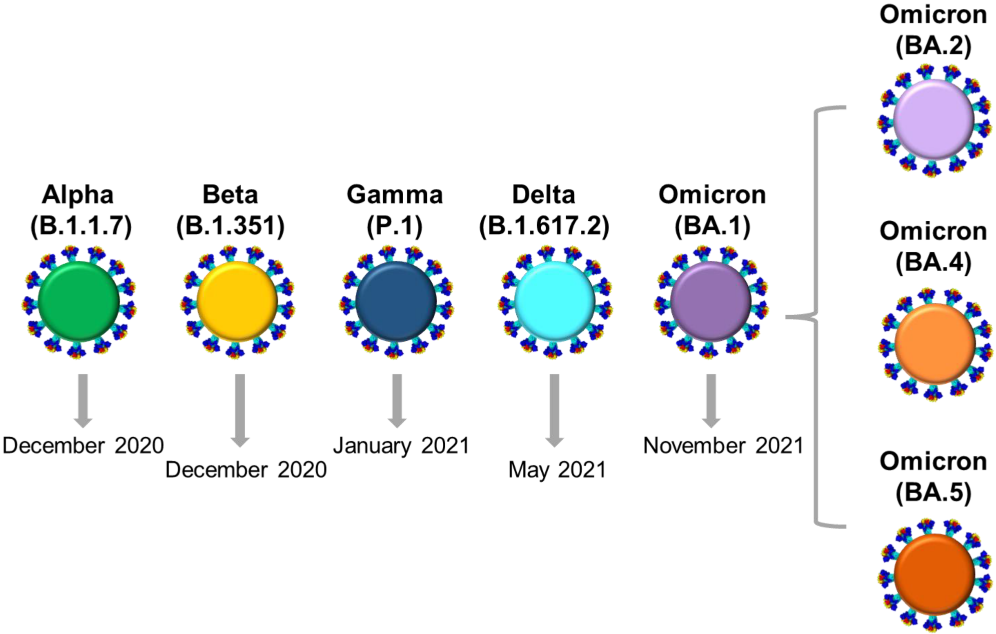
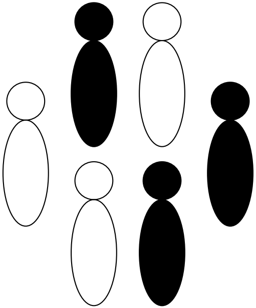
University of Cambridge

Upcoming talk:

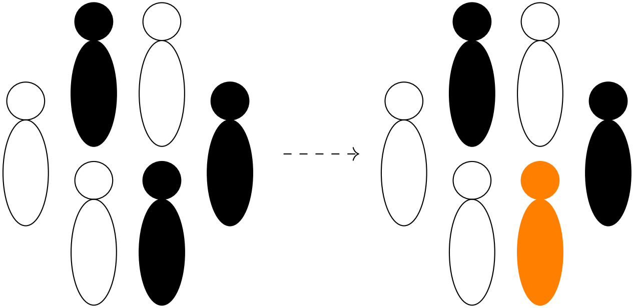
SMB MathEpiOnco 2024 (19/02 2:45pm-3:00pm EST).



Alpha
(B.1.1.7)
December 2020
Beta
(B.1.351)
December 2020
Gamma
(P.1)
January 2021
Delta
(B.1.617.2)
May 2021
Omicron
(BA.1)
November 2021
Omicron
(BA.2)
Omicron
(BA.4)
Omicron
(BA.5)

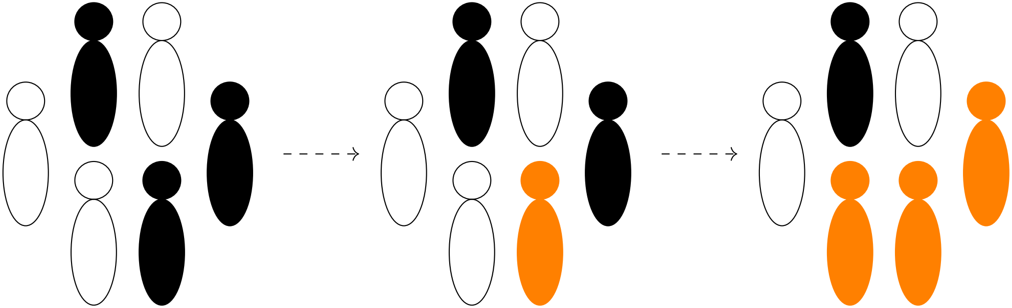


wildtype infections

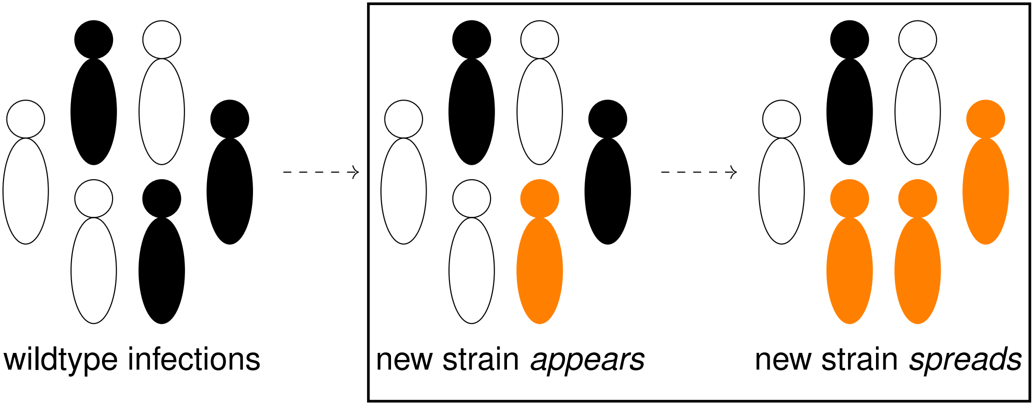


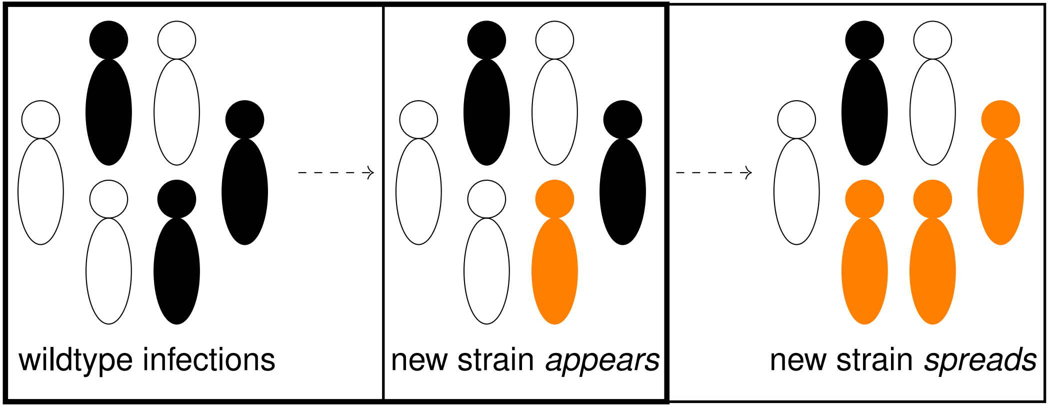
wildtype infections

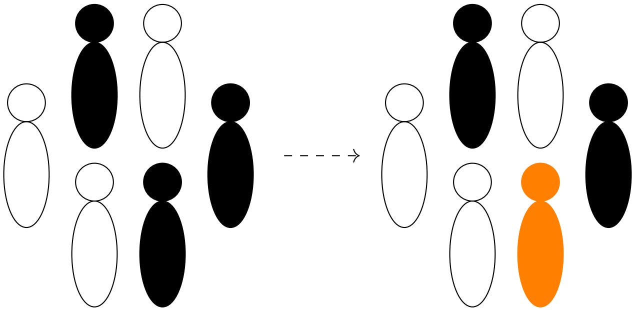
new strain *appears*

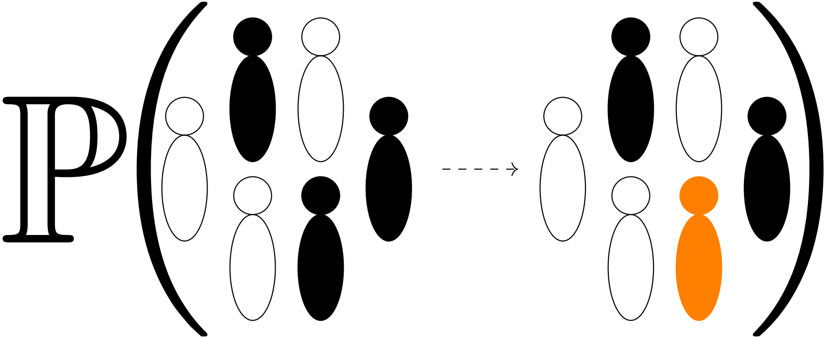


wildtype infections   new strain *appears*   new strain *spreads*


wildtype infections
new strain *appears*
new strain *spreads*

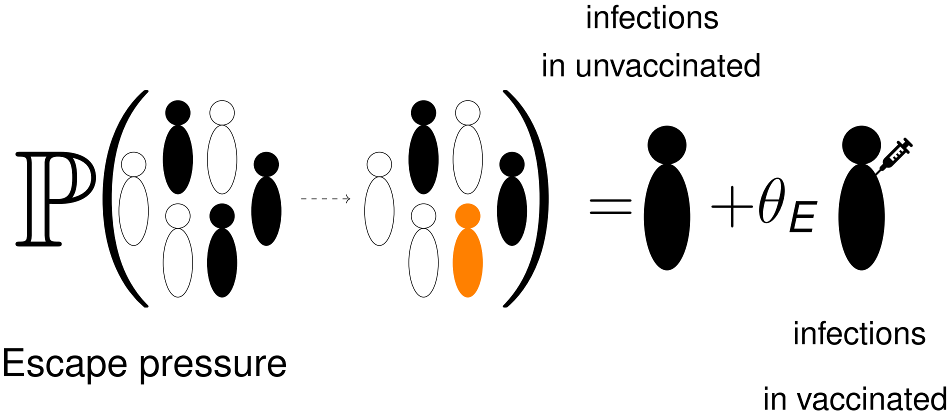

wildtype infections
new strain *appears*
new strain *spreads*



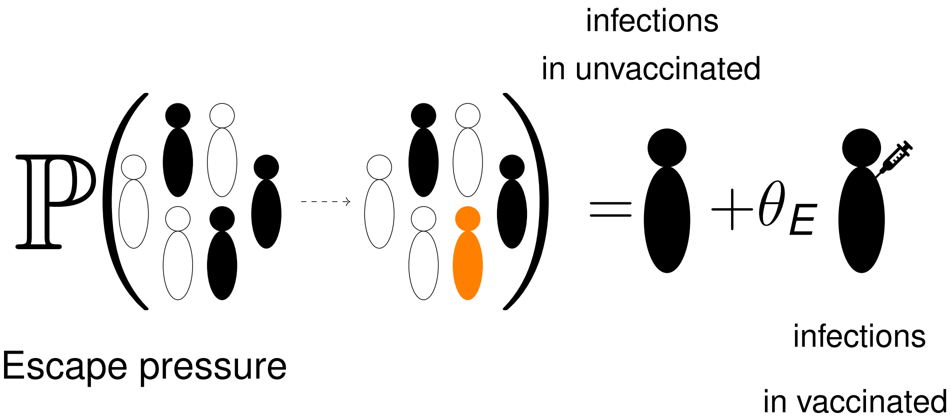


Escape pressure

Hartfield et al. 2014, Gog et al. 2021,
Rella et al. 2021, Saad-Roy et al. 2021

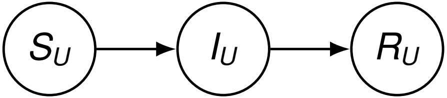


infections
in unvaccinated
$\mathbb{P}$
$= \ + \theta_E$
infections
in vaccinated
Escape pressure

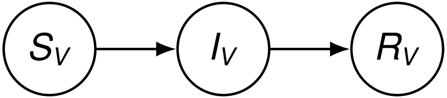


$$P(t) = I_U(t) + \theta_E I_V(t)$$

Unvaccinated fraction $(1-c)$



Vaccinated fraction c



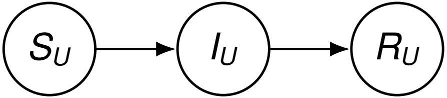
$$\dot{S}_U = -S_U \lambda(t) \qquad \dot{S}_V = -S_V \lambda(t)$$

$$\dot{I}_U = S_U \lambda(t) - I_U \qquad \dot{I}_V = S_V \lambda(t) - I_V$$

$$\lambda = R_0 (I_U + \theta_I I_V)$$

$$S_U(0) = (1-c) \qquad S_V(0) = c\theta_S$$

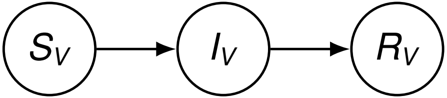
Unvaccinated fraction $(1-c)$



$$\dot{S}_U = -S_U\lambda(t)$$

$$\dot{I}_U = S_U\lambda(t) - I_U$$

Vaccinated fraction c



$$\dot{S}_V = -S_V\lambda(t)$$

$$\dot{I}_V = S_V\lambda(t) - I_V$$

$$\lambda = R_0(I_U + \theta_I I_V)$$

$$S_U(0) = (1-c) \qquad S_V(0) = c\theta_S$$

$$\forall t$$

$$(1-c)^{-1}(S_U, I_U, R_V) = (c\theta_S)^{-1}(S_V, I_V, R_V)$$

$$(S, I) := (1 - c)^{-1}(S_U, I_U) = (c\theta_S)^{-1}(S_V, I_V)$$

$$R_e = R_0(1 - c(1 - \theta_S\theta_I))$$

$$S(\infty) = -W\left(-R_e e^{-R_e}\right)/R_e \text{ (Lambert W-function)}$$

$$(S, I) := (1-c)^{-1}(S_U, I_U) = (c\theta_S)^{-1}(S_V, I_V)$$

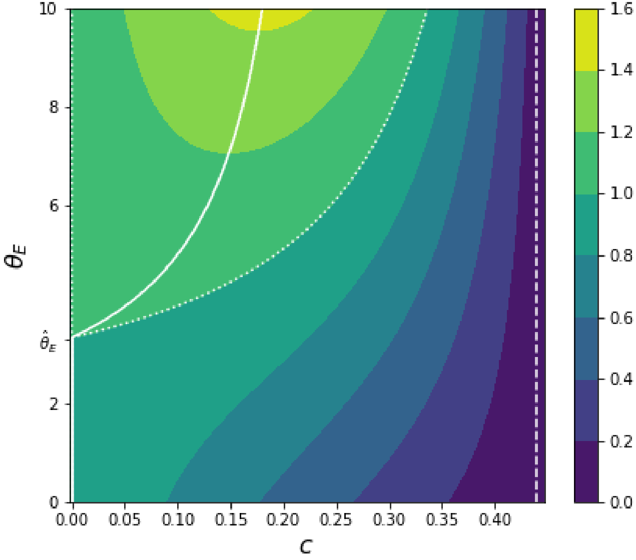
$$R_e = R_0(1 - c(1 - \theta_S\theta_I))$$

$S(\infty) = -W\left(-R_e e^{-R_e}\right)/R_e$ (Lambert W-function)

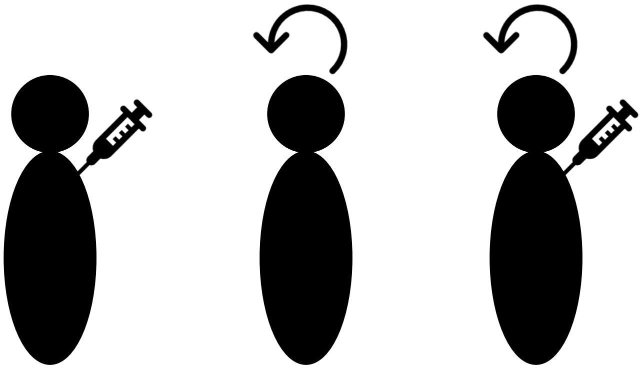
$$\int_0^\infty P(t)dt = \int_0^\infty (I_U + \theta_E I_V)dt$$

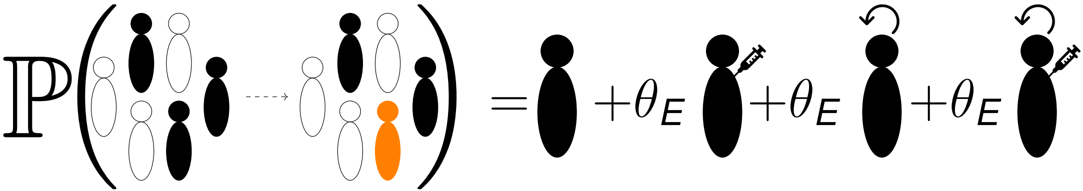
Cumulative escape pressure

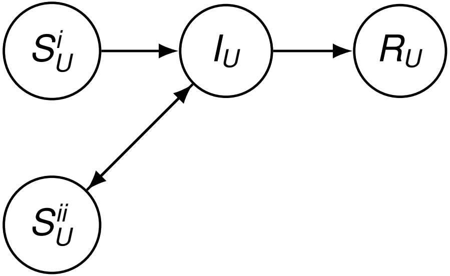
$$\boxed{P = (1 - c + \theta_S\theta_E c)\left(1 + \frac{1}{R_e}W\left(-R_e e^{-R_e}\right)\right)}$$

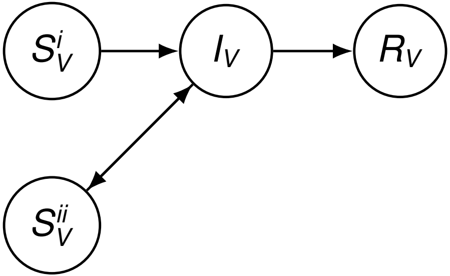


10
8
6
$\hat{\theta}_E$
2
0
θ_E
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
c
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0




$\mathbb{P}$
$= \; + \theta_E \; + \theta_E \; + \theta_E$





$$\dot{S}_U^i = -S_U^i \lambda(t)$$

$$\dot{S}_U^{ii} = -S_U^{ii} \lambda(t) + \theta'_S I_U$$

$$\dot{I}_U = (S_U^i + S_U^{ii})\lambda(t) - I_U$$

$$S_U^i(0) = 1 - c$$

$$\dot{S}_V^i = -S_V^i \lambda(t)$$

$$\dot{S}_V^{ii} = -S_V^{ii} \lambda(t) + \theta'_S I_V$$

$$\dot{I}_V = (S_V^i + S_V^{ii})\lambda(t) - I_V$$

$$S_U^{ii}(0) = c\theta_S$$

$$(S^i, S^{ii}, I) = (1-c)^{-1}(S^i_U, S^{ii}_U, I_U) = (c\theta_S)^{-1}(S^i_V, S^{ii}_V, I_V)$$

$$(S^i, S^{ii}, I) = (1-c)^{-1}(S^i_U, S^{ii}_U, I_U) = (c\theta_S)^{-1}(S^i_V, S^{ii}_V, I_V)$$

$$\frac{dS^{ii}}{dS^i} = \frac{\dot{S}^{ii}}{\dot{S}^i} = \ldots \implies \frac{d}{dS^i}\frac{S^{ii}}{S^i} = \cdots \implies$$

$$\frac{S^{ii}}{S^i} = \frac{\theta'_S}{R_e}(1/S^i - 1)$$

$$\frac{dI}{dS^i} = \frac{\dot{I}}{\dot{S}^i} = \cdots \implies$$

$$I = \left(1 - \frac{\theta'_S}{R_e}\right)(1 - S^i) + \frac{1 - \theta'_S}{R_e} \log S^i$$

$$\frac{dI}{dS^i} = \frac{\dot{I}}{\dot{S}^i} = \cdots \implies$$

$$I = \left(1 - \frac{\theta'_S}{R_e}\right)(1 - S^i) + \frac{1 - \theta'_S}{R_e} \log S^i$$

$$S^i_\infty = 1 - \frac{1 - \theta'_S}{R_e - \theta'_S} \log \frac{1}{S^i_\infty} = -\frac{1}{r} W[-re^{-r}]$$

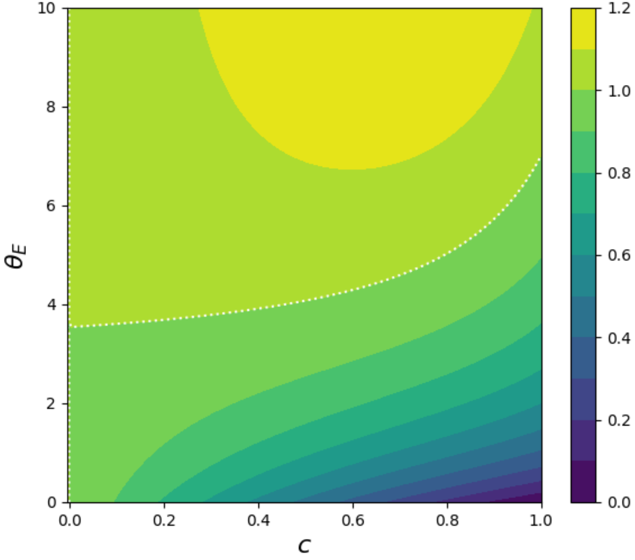
$$r = (R_e - \theta'_S)/(1 - \theta'_S)$$

$$\int_0^\infty R_e S^{ii} I dt = \int_{S^i_\infty}^1 \frac{S^{ii}}{S^i} dS^i$$
$$= \cdots = \frac{\theta'_S}{R_e}(r-1)\left(1 + \frac{1}{r} W[-re^{-r}]\right)$$

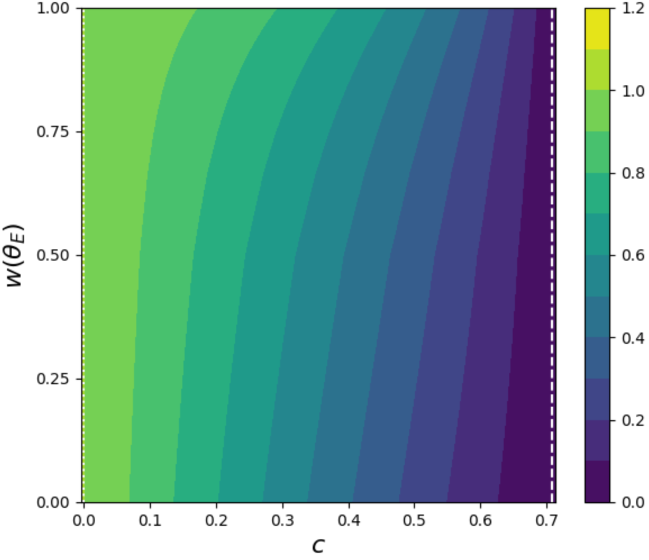
$$\int_0^\infty R_e S^{ii} I dt = \int_{S_\infty^i}^{1} \frac{S^{ii}}{S^i} dS^i$$

$$= \cdots = \frac{\theta'_S}{R_e}(r-1)\left(1 + \frac{1}{r} W[-re^{-r}]\right)$$

$$P = \left[1 - c + \theta_E \left(\frac{\theta'_S(1 - R_0^{-1}) + c(\theta_S - \theta'_S)}{1 - \theta'_S}\right)\right]\left(1 + \frac{W\left[-re^{-r}\right]}{r}\right)$$



10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0
θ_E
c
1.2
1.0
0.8
0.6
0.4
0.2
0.0



1.00
0.75
0.50
0.25
0.00
$w(\theta_E)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
c
1.2
1.0
0.8
0.6
0.4
0.2
0.0

Summary

Escape pressure

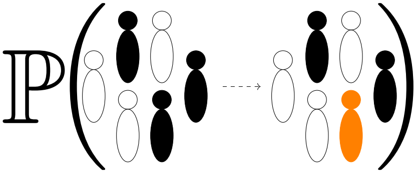
depends on the vaccination level:

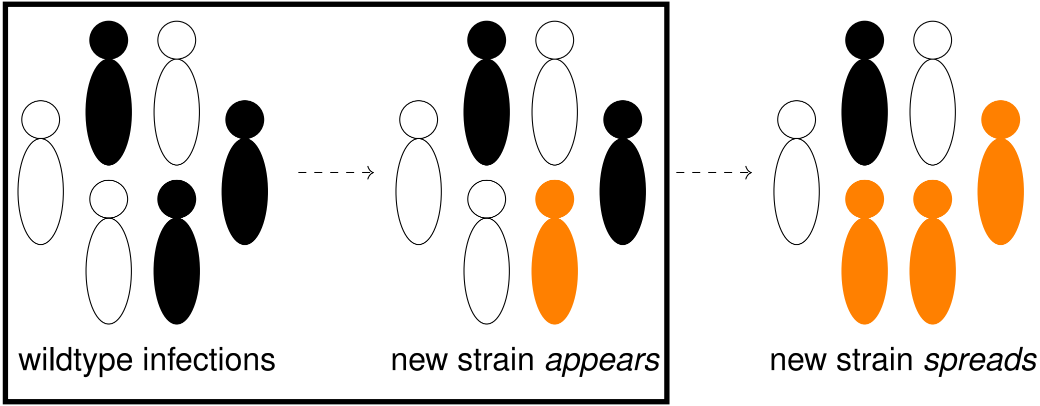
unimodal or decreasing

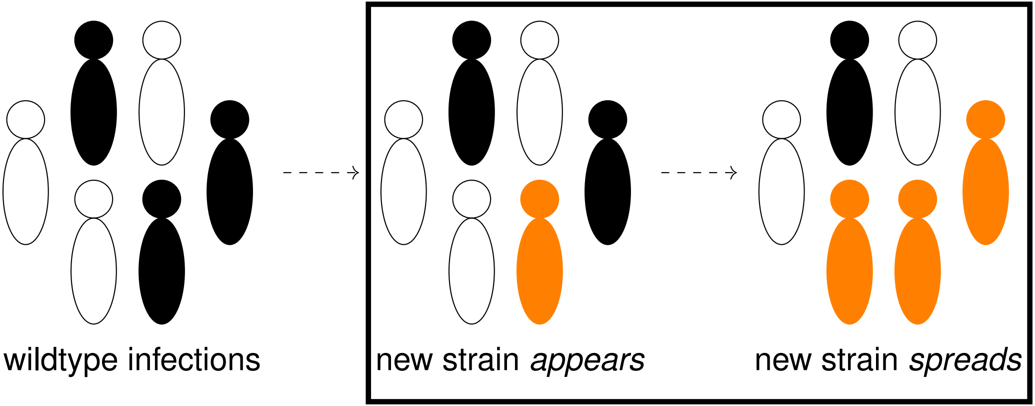
. . . determined by immunity

parameters and especially θ_E!

$\Longrightarrow$ **importance of heterogenous contributions to escape**




wildtype infections
new strain *appears*
new strain *spreads*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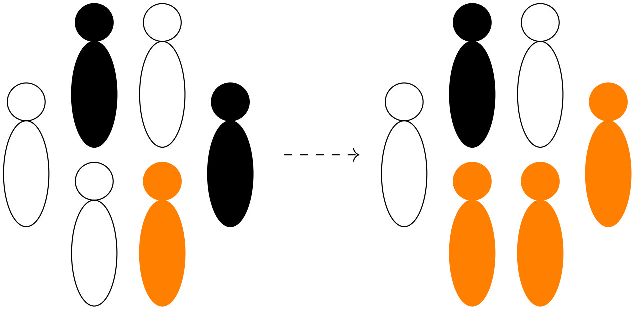
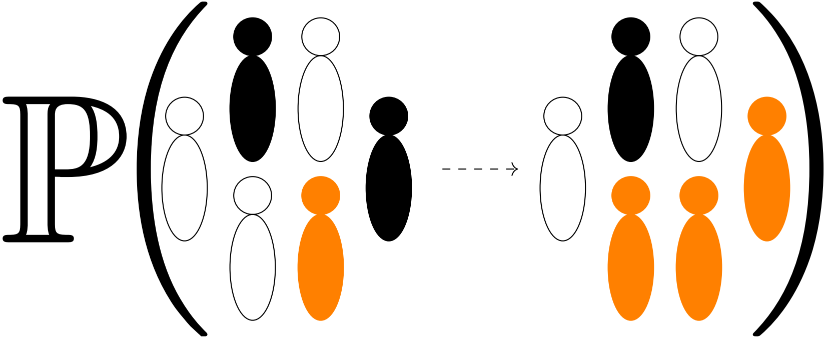
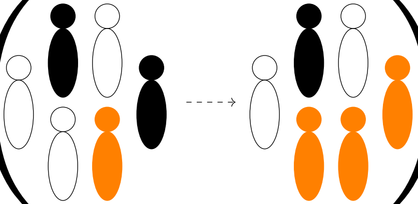
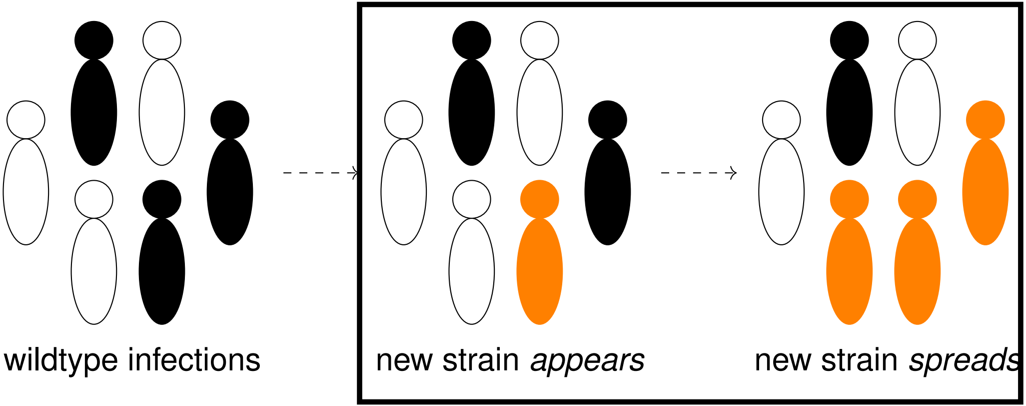




$\mathbb{P}\Bigg($  $\Bigg) \neq 1 - \frac{1}{R(t)}$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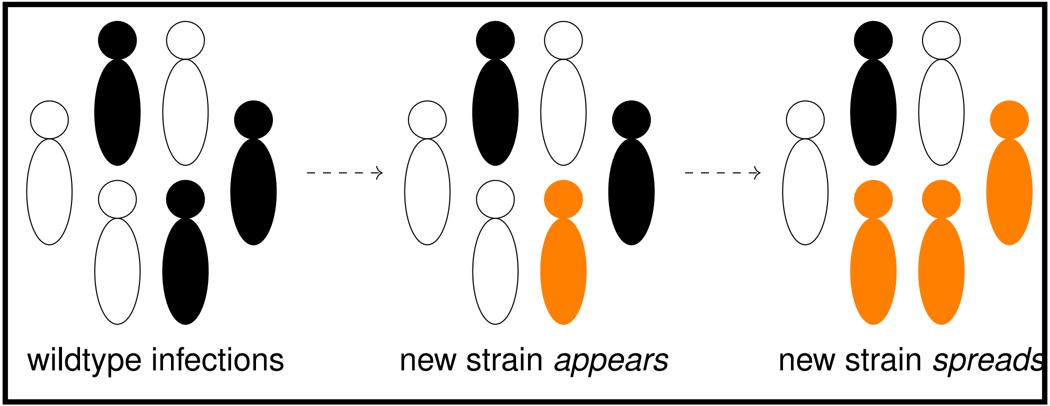
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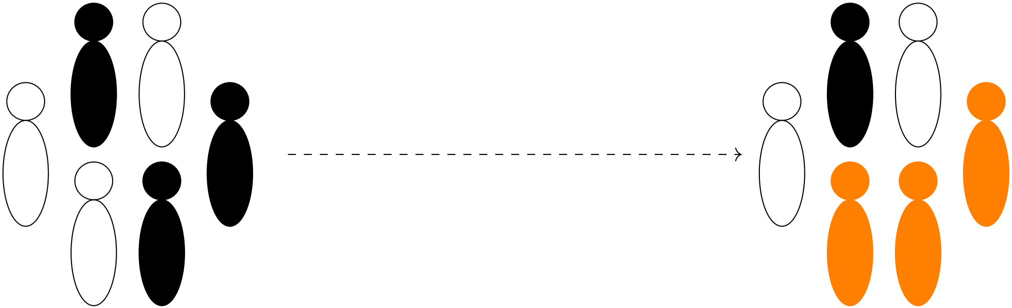

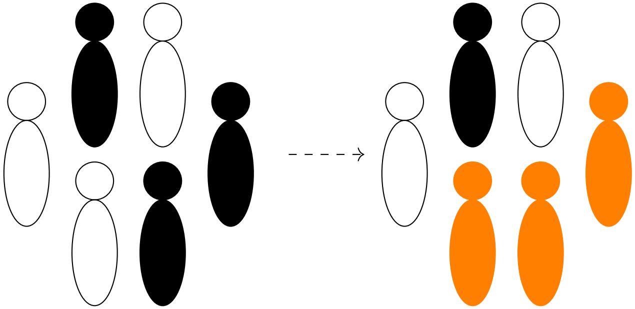

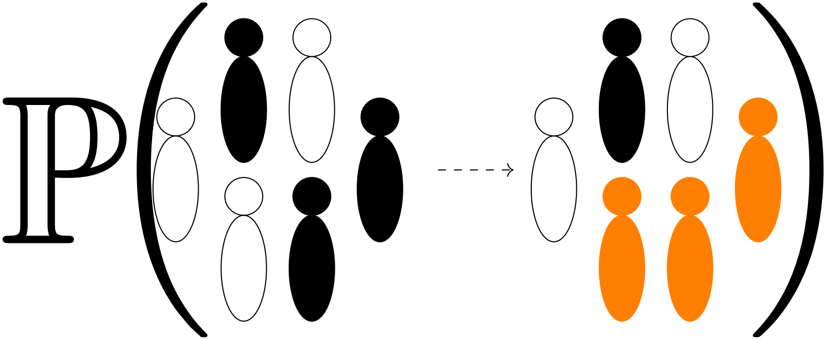

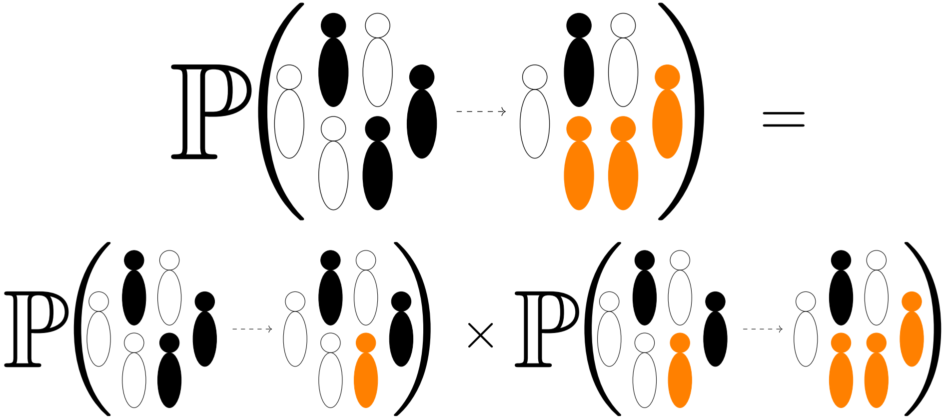

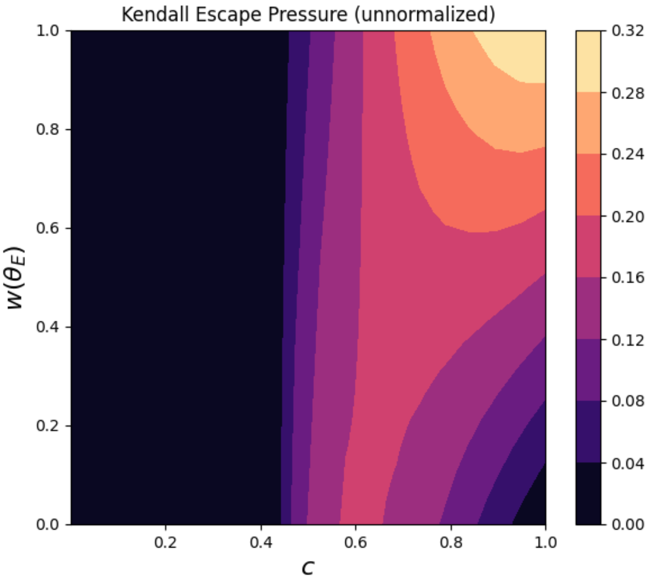

Kendall Escape Pressure (unnormalized)
$w(\theta_E)$
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
c
0.32
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0.00