

Vaccine Escape

Maria A. Gutierrez and Julia R. Gog

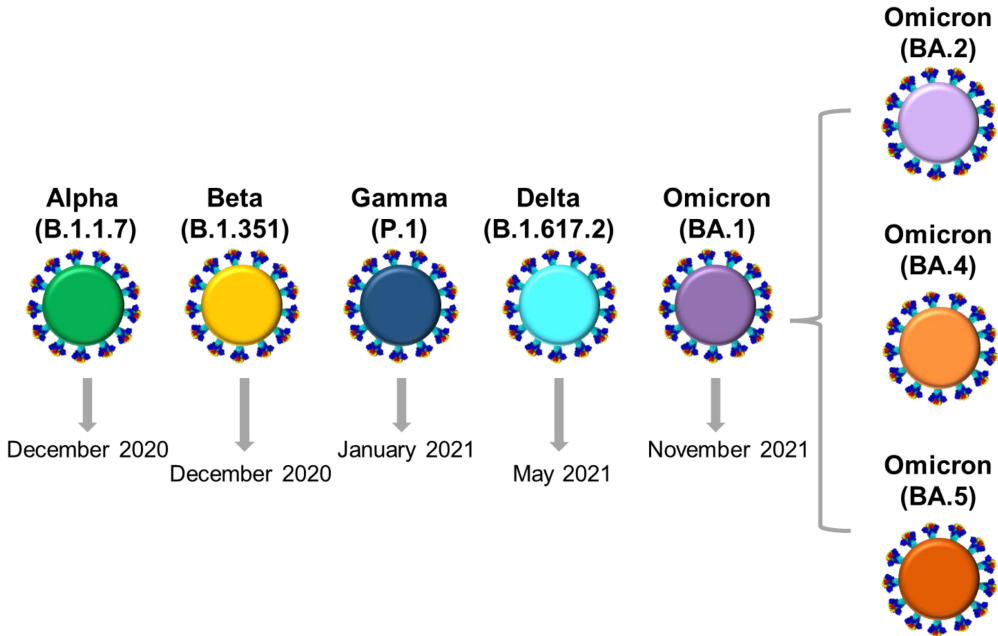
University of Cambridge

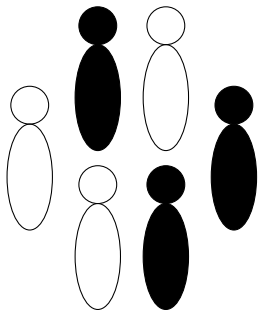


Upcoming talk:

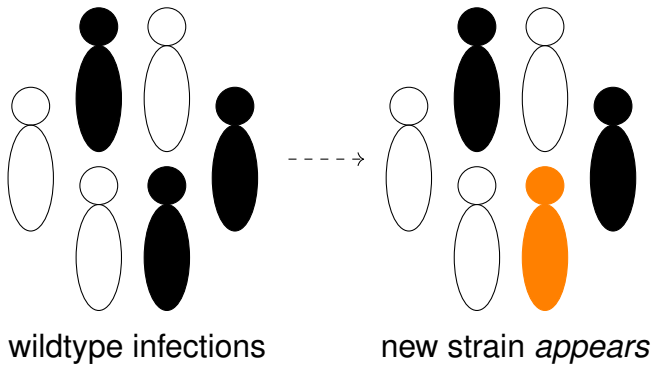
SMB MathEpiOnco 2024 (19/02 2:45pm-3:00pm EST).

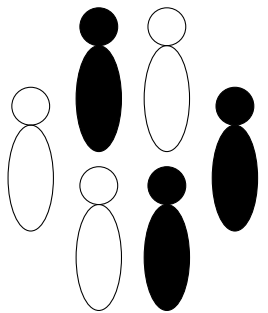




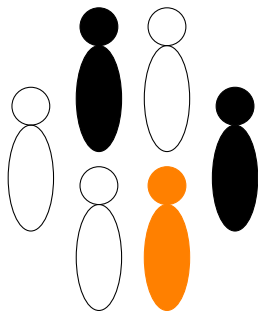


wildtype infections

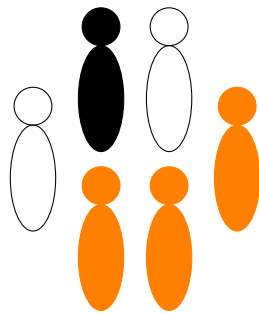




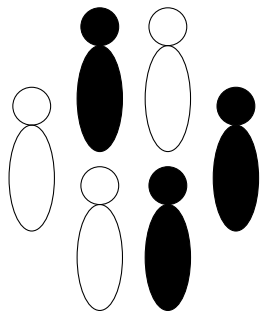
wildtype infections



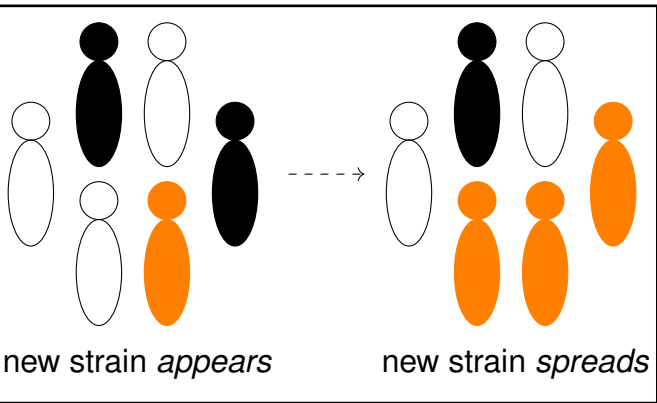
new strain *appears*



new strain *spreads*

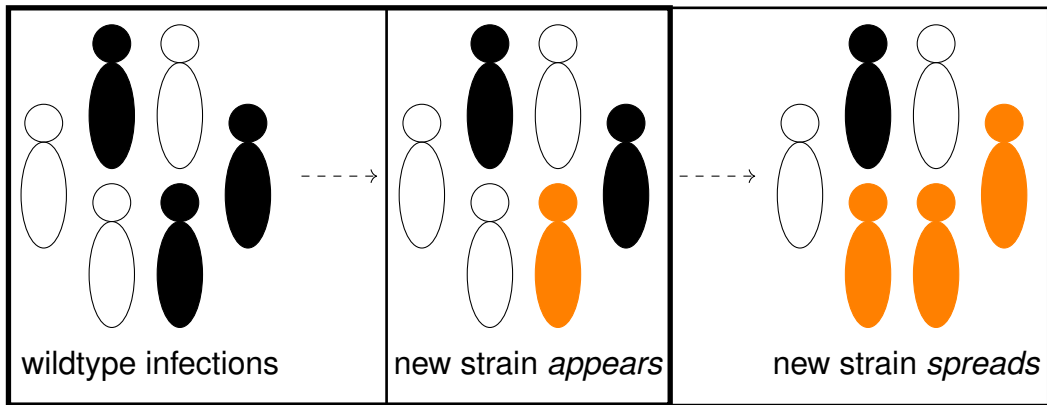


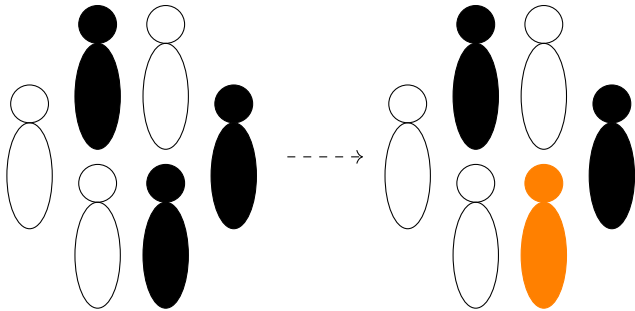
wildtype infections

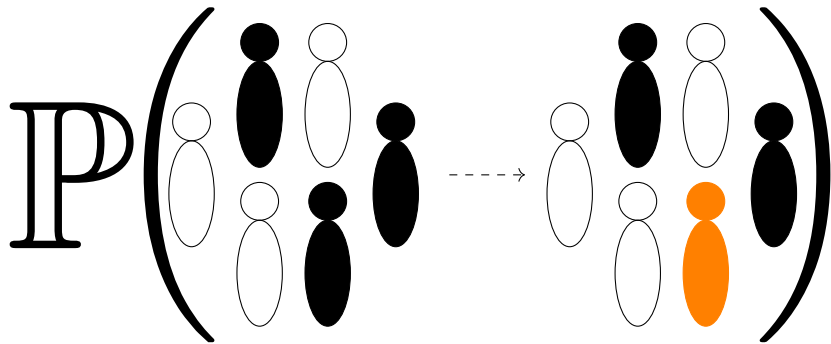


new strain *appears*

new strain *spreads*







Escape pressure

Hartfield et al. 2014, Gog et al. 2021,
Rella et al. 2021, Saad-Roy et al. 2021

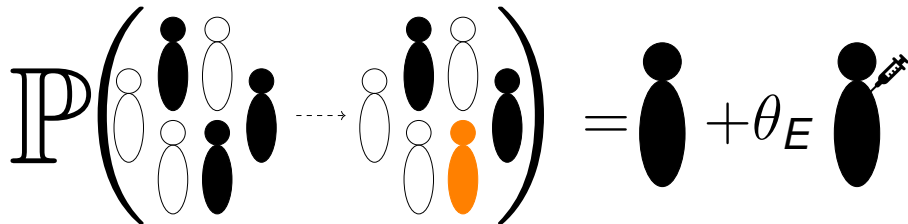
infections
in unvaccinated



Escape pressure

infections
in vaccinated

infections
in unvaccinated



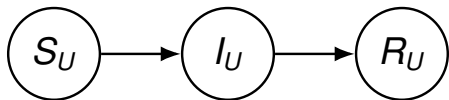
Escape pressure

infections

in vaccinated

$$P(t) = I_U(t) + \theta_E I_V(t)$$

Unvaccinated fraction $(1 - c)$

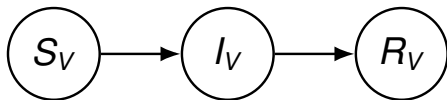


$$\dot{S}_U = -S_U\lambda(t)$$

$$\dot{I}_U = S_U\lambda(t) - I_U$$

$$S_U(0) = (1 - c)$$

Vaccinated fraction c



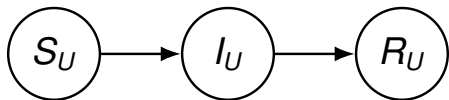
$$\dot{S}_V = -S_V\lambda(t)$$

$$\dot{I}_V = S_V\lambda(t) - I_V$$

$$S_V(0) = c\theta_s$$

$$\lambda = R_0(I_U + \theta_I I_V)$$

Unvaccinated fraction $(1 - c)$

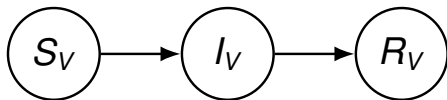


$$\dot{S}_U = -S_U\lambda(t)$$

$$\dot{I}_U = S_U\lambda(t) - I_U$$

$$S_U(0) = (1 - c)$$

Vaccinated fraction c



$$\dot{S}_V = -S_V\lambda(t)$$

$$\dot{I}_V = S_V\lambda(t) - I_V$$

$$S_V(0) = c\theta_S$$

$$\lambda = R_0(I_U + \theta_I I_V)$$

$\forall t$

$$(1 - c)^{-1}(S_U, I_U, R_U) = (c\theta_S)^{-1}(S_V, I_V, R_V)$$

$$(S, I) := (1 - c)^{-1}(S_U, I_U) = (c\theta_S)^{-1}(S_V, I_V)$$

$$R_e = R_0(1 - c(1 - \theta_S\theta_I))$$

$$S(\infty) = -W(-R_e e^{-R_e}) / R_e \text{ (Lambert W-function)}$$

$$(S, I) := (1 - c)^{-1}(S_U, I_U) = (c\theta_S)^{-1}(S_V, I_V)$$

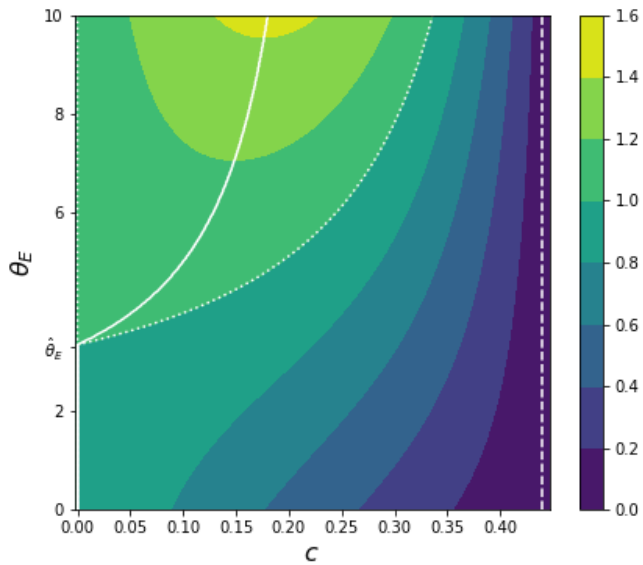
$$R_e = R_0(1 - c(1 - \theta_S\theta_I))$$

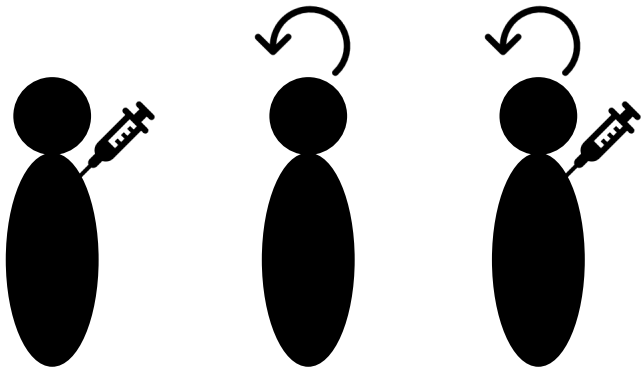
$$S(\infty) = -W(-R_e e^{-R_e}) / R_e \text{ (Lambert W-function)}$$

$$\int_0^\infty P(t) dt = \int_0^\infty (I_U + \theta_E I_V) dt$$

Cumulative escape pressure

$$P = (1 - c + \theta_S\theta_EC) \left(1 + \frac{1}{R_e} W(-R_e e^{-R_e}) \right)$$

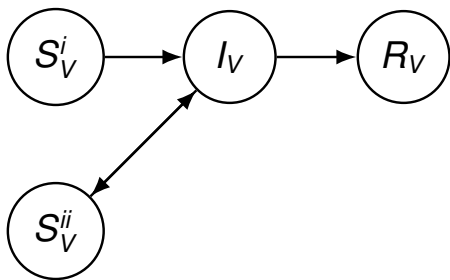
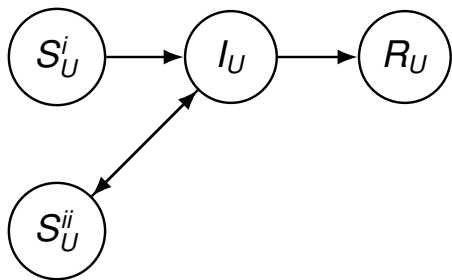




$$\mathbb{P} \left(\begin{array}{c} \text{[Group of 8 people: 4 white, 4 black]} \\ \text{---} \\ \text{[Group of 8 people: 5 white, 1 orange, 2 black]} \end{array} \right) = \text{[1 black person]} + \theta_E \text{[1 black person with syringe]}$$

The diagram illustrates a probability distribution over a population of 8 people. On the left, a large set of parentheses contains two states of the population, separated by a dashed arrow. The top state shows 4 white and 4 black people. The bottom state shows 5 white, 1 orange, and 2 black people. To the right of the parentheses is an equals sign followed by a single black person icon, plus a coefficient θ_E multiplied by a black person icon with a syringe.

$$\mathbb{P} \left(\begin{array}{c} \text{[Group of 7 people: 2 white, 3 black, 2 black]} \\ \text{--->---} \\ \text{[Group of 7 people: 2 white, 1 black, 1 orange, 3 black]} \end{array} \right) = \text{[1 black]} + \theta_E \text{[1 black with syringe]} + \theta_E \text{[1 black with arrow]} + \theta_E \text{[1 black with syringe and arrow]}$$



$$\dot{S}_U^i = -S_U^i \lambda(t)$$

$$\dot{S}_U^{ii} = -S_U^{ii} \lambda(t) + \theta'_S I_U$$

$$\dot{I}_U = (S_U^i + S_U^{ii}) \lambda(t) - I_U$$

$$S_U^i(0) = 1 - c$$

$$\dot{S}_V^i = -S_V^i \lambda(t)$$

$$\dot{S}_V^{ii} = -S_V^{ii} \lambda(t) + \theta'_S I_V$$

$$\dot{I}_V = (S_V^i + S_V^{ii}) \lambda(t) - I_V$$

$$S_U^{ii}(0) = c \theta_S$$

$$(S^i, S^{ii}, I) = (1-c)^{-1}(S_U^i, S_U^{ii}, I_U) = (c\theta_S)^{-1}(S_V^i, S_V^{ii}, I_V)$$

$$(S^i, S^{ii}, I) = (1-c)^{-1}(S^i_U, S^{ii}_U, I_U) = (c\theta_S)^{-1}(S^i_V, S^{ii}_V, I_V)$$

$$\frac{dS^{ii}}{dS^i} = \frac{\dot{S}^{ii}}{\dot{S}^i} = \dots \implies \frac{d}{dS^i} \frac{S^{ii}}{S^i} = \dots \implies$$

$$\frac{S^{ii}}{S^i} = \frac{\theta'_S}{R_e} (1/S^i - 1)$$

$$\frac{dl}{dS^i} = \frac{\dot{l}}{\dot{S}^i} = \dots \implies$$

$$l = \left(1 - \frac{\theta'_S}{R_e}\right) (1 - S^i) + \frac{1 - \theta'_S}{R_e} \log S^i$$

$$\frac{dl}{dS^i} = \frac{\dot{l}}{\dot{S}^i} = \dots \implies$$

$$l = \left(1 - \frac{\theta'_S}{R_e}\right) (1 - S^i) + \frac{1 - \theta'_S}{R_e} \log S^i$$

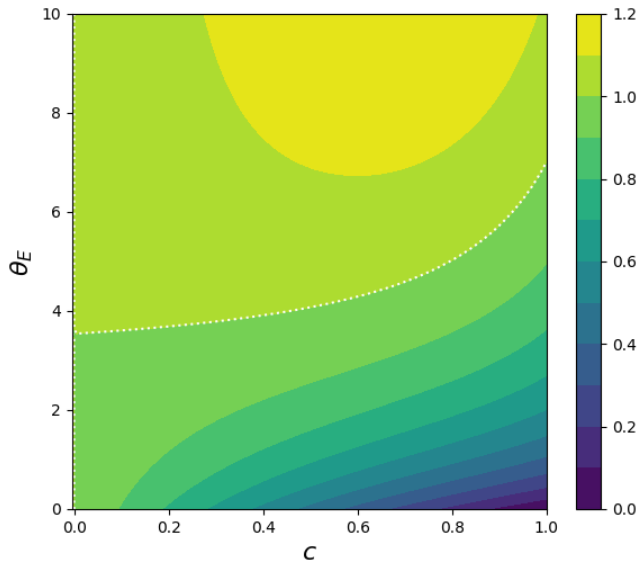
$$S^i_\infty = 1 - \frac{1 - \theta'_S}{R_e - \theta'_S} \log \frac{1}{S^i_\infty} = -\frac{1}{r} W[-re^{-r}]$$

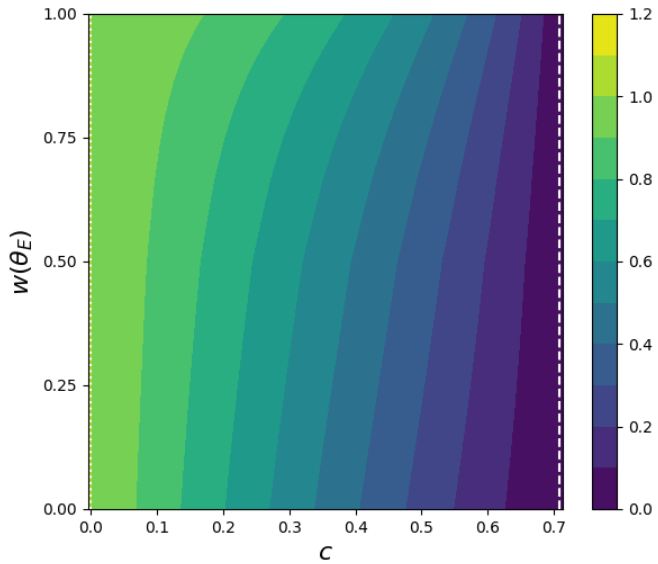
$$r = (R_e - \theta'_S)/(1 - \theta'_S)$$

$$\begin{aligned}\int_0^\infty R_e S^{ii} I dt &= \int_{S_\infty^i}^1 \frac{S^{ii}}{S^i} dS^i \\ &= \dots = \frac{\theta'_S}{R_e} (r - 1) \left(1 + \frac{1}{r} W[-re^{-r}] \right)\end{aligned}$$

$$\begin{aligned}
 \int_0^\infty R_e S^{ii} I dt &= \int_{S_\infty^i}^1 \frac{S^{ii}}{S^i} dS^i \\
 &= \dots = \frac{\theta'_S}{R_e} (r - 1) \left(1 + \frac{1}{r} W[-re^{-r}] \right)
 \end{aligned}$$

$$P = \left[1 - c + \theta_E \left(\frac{\theta'_S (1 - R_0^{-1}) + c(\theta_S - \theta'_S)}{1 - \theta'_S} \right) \right] \left(1 + \frac{W[-re^{-r}]}{r} \right)$$





Summary

Escape pressure

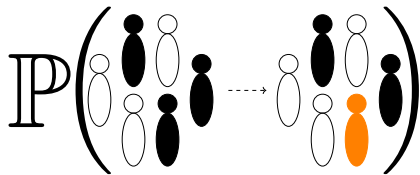
depends on the vaccination level:

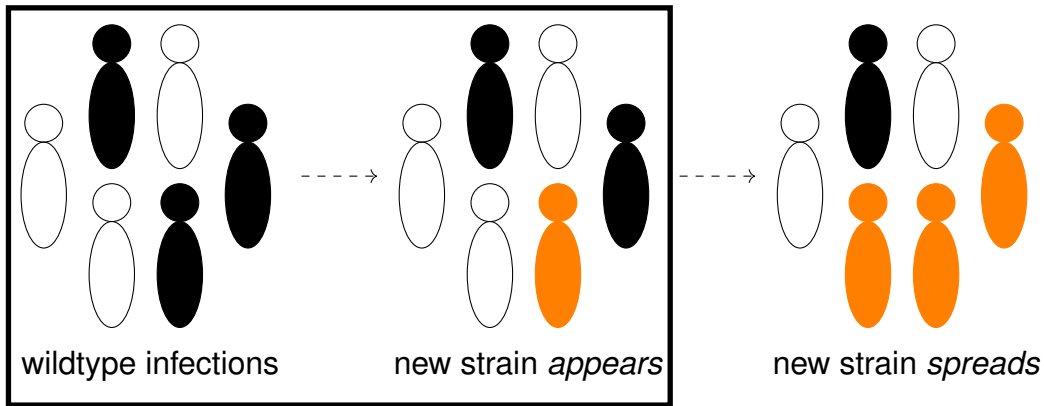
unimodal or decreasing

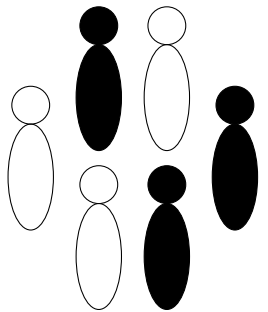
... determined by immunity

parameters and especially θ_E !

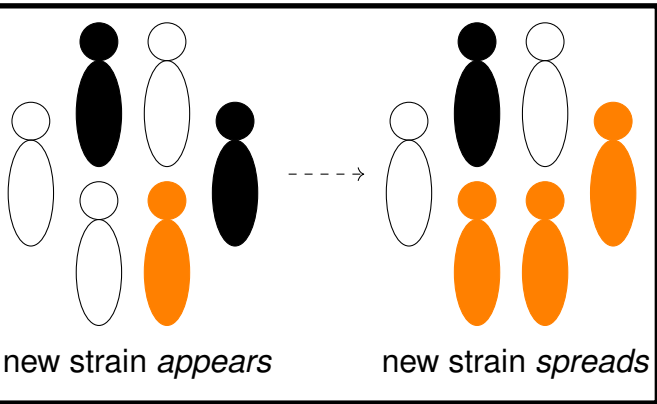
⇒ importance of heterogeneous contributions to escape





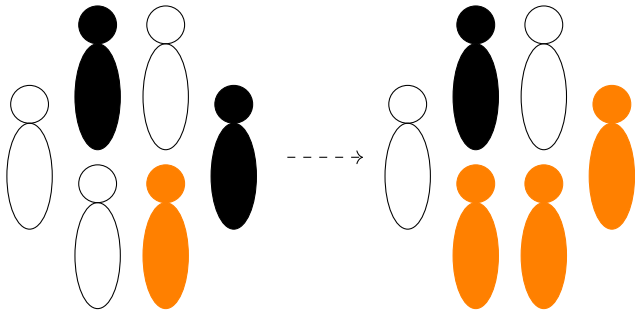


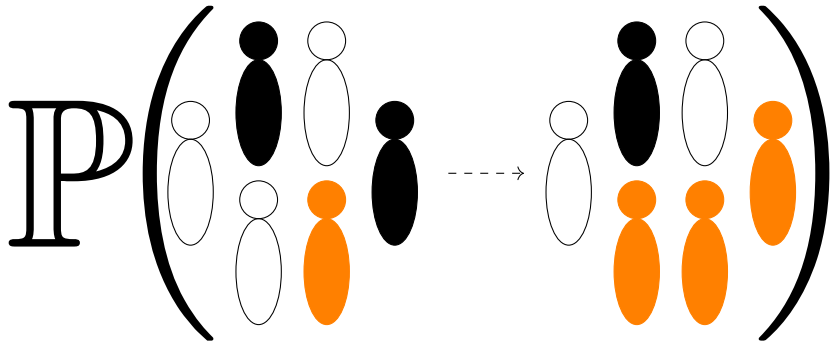
wildtype infections



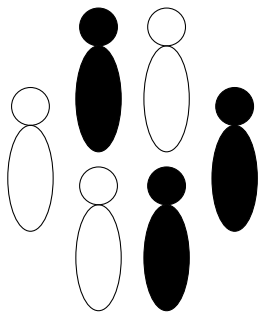
new strain *appears*

new strain *spreads*

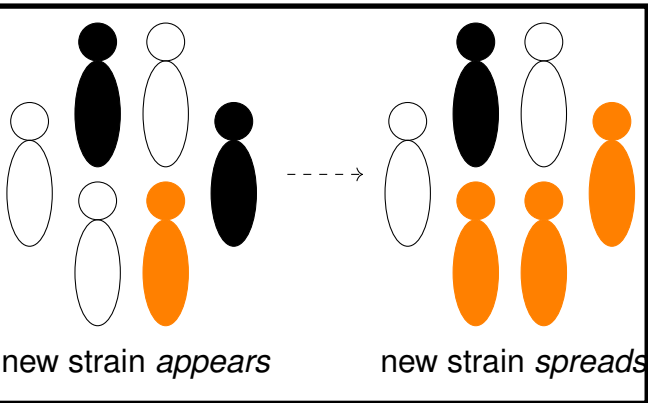




$$\mathbb{P}\left(\begin{array}{c} \text{[Diagram: Initial state with 7 people: 2 white, 2 black, 1 orange, 1 black]} \\ \text{---} \\ \text{[Diagram: Final state with 7 people: 1 white, 1 black, 1 white, 3 orange]} \end{array}\right) \neq 1 - \frac{1}{R(t)}$$

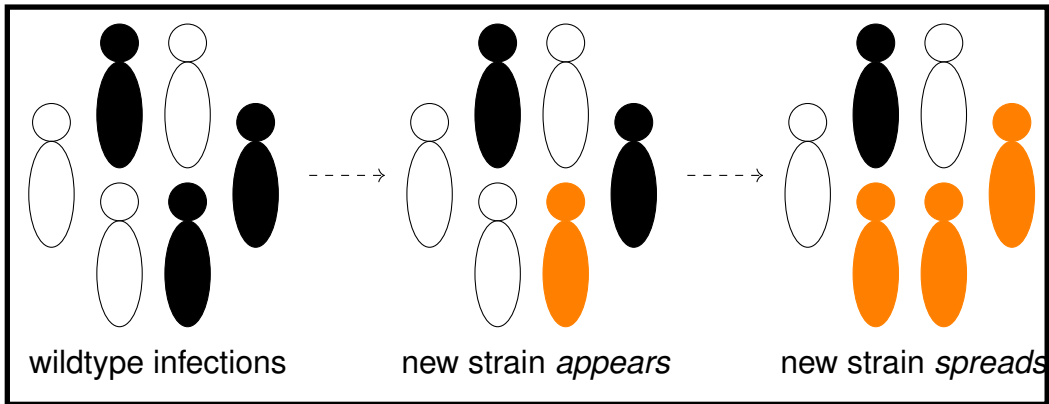


wildtype infections

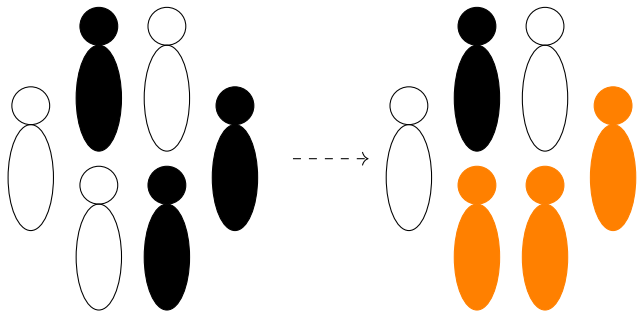


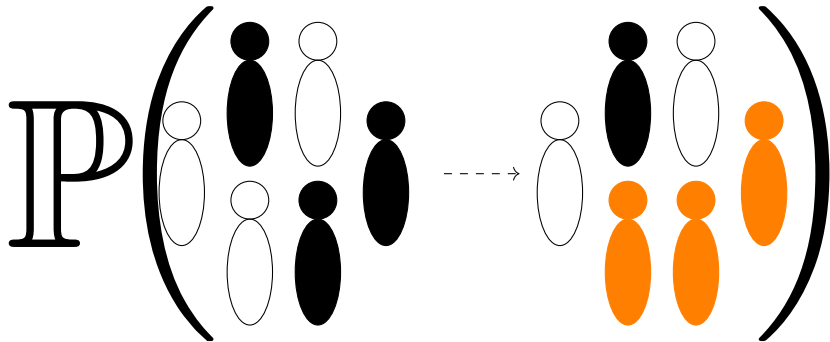
new strain *appears*

new strain *spreads*









$$\mathbb{P} \left(\begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \end{array} \rightarrow \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \end{array} \right) = \mathbb{P} \left(\begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \end{array} \rightarrow \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Orange} \end{array} \right) = \\
 \mathbb{P} \left(\begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \end{array} \rightarrow \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Orange} \end{array} \right) \times \mathbb{P} \left(\begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Black} \end{array} \rightarrow \begin{array}{c} \text{Black} \quad \text{White} \\ \text{White} \quad \text{Orange} \end{array} \right)$$

